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PHYSICS

By

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PREFACE

This book has been written to assist students of two types. The first is the student in school or technical college who wishes to study heat, light and sound in a way which is closely co-ordinated with practical applications and made real by illustration from everyday life. The second type is the student who realises that a knowledge of physics would assist him or her in present or future work and finds it necessary to study with little or no help from a tutor. This latter class deserves every encouragement and no effort has been spared to produce a book which would be of real help. No mathematical knowledge beyond simple arithmetic has been assumed and typical worked examples of problems are included where it was thought desirable to make the application clear. Detailed instructions are given for the conduct of experiments. Most of these can be carried out with 'home-made' apparatus.

It is a pleasure to record the extremely valuable assistance given by Dr. E. G. Richardson of King's College, Newcastle-upon-Tyne, University of Durham. The drawings have been executed by Mr. Alan Richardson.

Acknowledgement is also made to Messrs. Gallenkamp of London for the loan of blocks of Figures 2, 13 and 69.

I am indebted to those who have sent me suggestions for the greater usefulness of this book.

W. RAILSTON.

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Light.

Happy the Man,
Who studying Nature's laws,
Through known effects can
Trace the secret cause.

VIRGIL: *Georgics*, Book II.
(Dryden's Translation).

SECTION ON MEASUREMENTS AND UNITS

CHAPTER I

PRELIMINARY IDEAS

The Scope of Physics

Physics properly deals with the behaviour of all material not animated by vital force or 'self-will'. From its comprehensive scope, dealing as it does with the fundamental laws of nature, it was, and is still in some of the older universities, known as Natural Philosophy. As a matter of convenience and by common agreement, the range of a course in physics is further restricted. Thus though the animal and vegetable worlds react on their environment through the laws of physics, this branch (biophysics) is usually included in a biological course, while those aspects which concern the natural or artificial manipulation of matter on a large scale are appropriated to geology or engineering. This narrows the field to physics and chemistry, subjects which overlap to a considerable extent.* Of these we may say that the latter embraces changes which endow the molecules of a substance or series of substances with completely new properties. To give border-line instances, the freezing of water is regarded as a physical process, but the production of smoke when ammonia and hydrochloric acid vapours mix belongs to chemistry.

Physics is essentially a science of exact measurement, though, like all science, it began as a series of disconnected observations. Inasmuch as every branch of science needs the instruments used in physics to make its measurements, physics may be said to be the most fundamental and important of all the sciences. Every scientific study advances

* There are periodicals devoted to 'physical chemistry' and to 'chemical physics'.

from qualitative to quantitative observation, from natural history to applied physics.

The Branches of Physics

Physics itself is divided into branches, for convenience, though here again there is a certain amount of overlap. We may illustrate these by reference to the electric spark, which under suitable circumstances may be made to jump the short distance between two metal knobs placed close together in air or—on a larger scale—between a cloud and the earth. The spark itself produces **light**, perceived by the eye, while the crackling—or thunder, in the corresponding natural phenomenon—is a **sound**, perceived by the ear. Beside this the air in the vicinity is warmed. The hand placed near the knobs after the discharge tells one that **heat** has been produced. At the same time the source of the spark is a difference of potential between the knobs, or between the cloud and the earth, and is **electrical**. The facts that the electrodes may be eaten away by a succession of such sparks and that a current of air is set up by them are **properties of the matter** in the surroundings, and would be studied under the head of **mechanics**. (This covers the physical aspects of the 'electric spark', though it also has chemical and biological associations. The smell which accompanies the spark is due to the changing of some of the oxygen in the air to ozone; while if one holds a knob in each hand at the instant of discharge, an involuntary contraction of one's muscles takes place.)

The words in bold type in the above paragraph indicate the main branches into which physics is conventionally divided. It should be explained at the outset that the author does not intend to deal with mechanics or electricity in this book, except incidentally. This quite artificial restriction of the term 'physics' to Heat, Light, and Sound and some of the relevant Properties of Matter is occasioned by the fact that there are already a 'Teach Yourself Mechanics' and a 'Teach Yourself Electricity' in this series. Though this little book is complete in itself, the reader is recommended to read these if he has not

already studied these subjects. Particularly, in mechanics, he should make himself familiar with the meaning of the terms 'force, work, and energy'.

Methods of Measurement

Physics, as we have emphasized, is an exact science, and an exact science cannot exist without accurate measurement. In order that measurements made at one place may be recorded and reproduced at a later time or in another place, standards of measurement are necessary. These are kept at certain capitals; they are the originals of the two systems commonly in use in London and in Paris respectively, and from them copies are made for use elsewhere. The standards of the metric system—the one most commonly employed in physical measurements—originally bore an arbitrary relationship to the circumference of the earth, but whether they in fact still bear this relationship is beside the point, as long as physicists agree to abide by them.

Units; Fundamental and Derived

If it were necessary to keep a standard for every type of quantity that one can measure in science, the bureaux of standards belonging to each nation would become large-sized museums. In fact, it is necessary to have standards of quite a limited number of *fundamental quantities*, as they are called, from which other standards are *derived*. In mechanics and sound only three such standards are necessary —viz., those of **length, mass, and time.** We derive a standard of velocity, for example, from those of length and time, since it is measured by the distance covered by the moving body in a certain time; of acceleration, as a change of velocity in a certain time; of force, as the product of mass and acceleration. When we come to heat, however, we find that another standard is necessary, that for **temperature,** something to express the degree of hotness or coldness which we experience when we come in contact with other bodies. It is not possible to express this idea in terms of length, mass, or time. (The fact that tempera-

ture is often *measured* as a length or change of length—see below—does not affect the statement that temperature, itself, is not a length. We cannot state that a glowing lamp is 'three yards hot'!). A standard of temperature must accordingly be added to those of length, mass, and time in the study of heat, and this leads to further derived quantities which the reader will meet in that section. Two more fundamental standards must be introduced in which to express measurements made in magnetism and electricity.

Measurements, then, will be expressed in terms of

(1) The British system, of which the standards are: the *yard*, the *pound avoirdupois*, the *second*, and the *degree Fahrenheit*, or of

(2) The metric system, of which the standards are: the *metre*, the *kilogram*, the *second*, and the *degree Centigrade*,

in the order of the quantities measured above.

A unit of measurement need not be the same size as the standard, as long as it possesses an exact relationship to it. Usually it is a submultiple; for example, the *foot* as a third of a yard, the *centimetre* as a hundredth of a metre, the *gram* as a thousandth of a kilogram, etc.

(It is desirable not to fall into the habit of using bastard dimensions formed of fundamental units chosen from the two systems indiscriminately. Such derived units as 'kilograms per square foot', 'pound degree Centigrade' are as confusing as a series of measurements made during an experiment partly in inches and partly in centimetres.)

Measurement of Length

The object whose length is required is placed alongside a scale, which is inscribed in suitable fractions of the standard, bearing in mind the accuracy which is expected to be attained. If for instance it is a tennis lawn which is being mapped out, it will be sufficient to have a tape marked in yards or metres, but to measure the diameter of a ball-bearing by calipers the scale will need to be graduated in

thirty-seconds of an inch or in millimetres. The zero of the scale is placed as nearly opposite the one end of the object as the eye can judge, while the eye notes which mark on the scale falls most nearly opposite to the other end. Often it is possible to judge the fraction of a division by which the object overhangs the next lowest mark on the scale; with the measuring tape marked in feet, for instance, to judge a quarter or half a foot extra, or a fifth of a millimetre in measuring the gap between the caliper jaws. Physicists prefer, however, wherever possible to ask of the senses no more than that they should be able to judge a 'coincidence'—of the eye, when two marks are exactly opposite; of the ear, when two sounds occur simultaneously.

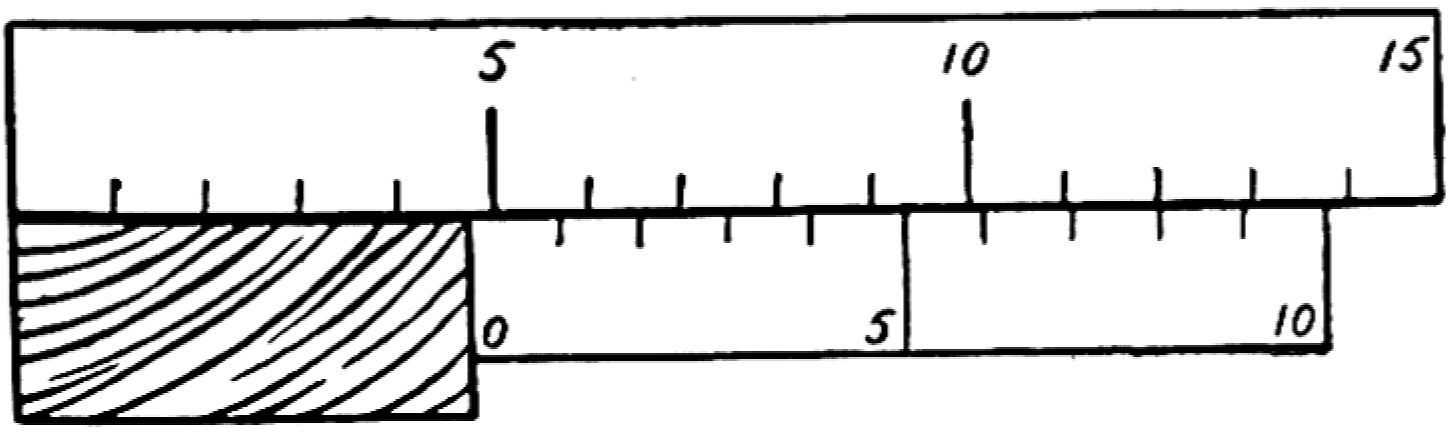


FIG. 1.—SCALE WITH VERNIER.

To carry out this idea in the measurement of length, a vernier is used. This (Fig. 1) is an additional short scale capable of sliding alongside or partly over the main scale. On a metric scale the vernier has ten divisions, which altogether cover nine of those of the main scale. If the main scale is marked in millimetres, each division of the vernier will be nine-tenths of a millimetre long. The zero of the vernier scale (which is actually the edge of it) is pushed up against the end of the object (which in the figure lies between 4 and 5 mm. on the main scale), and one looks along to see which mark on the vernier scale—the 8 in the figure—'coincides with' (lies exactly opposite) a division of the main scale. Therefore graduation 7 on the vernier scale lies 0·1 mm. to the right of the next mark to the left of the 'coincidence' on the main scale, graduation 6 lies 0·2 mm.

beyond, etc., and the zero of the vernier scale lies 0·8 mm. beyond the division 4 of the main scale. Thus the object is 4·8 mm. long—in other words, in the measure of the object the division of the vernier scale which has a coincidence with one of the main scale gives the next decimal point, as a fraction of a millimetre.

Measurement of Mass

A series of *weights* in a box are provided, and comparison of the 'unknown mass' with the nearest one of these is commonly made on a balance in the form of an arm pivoted

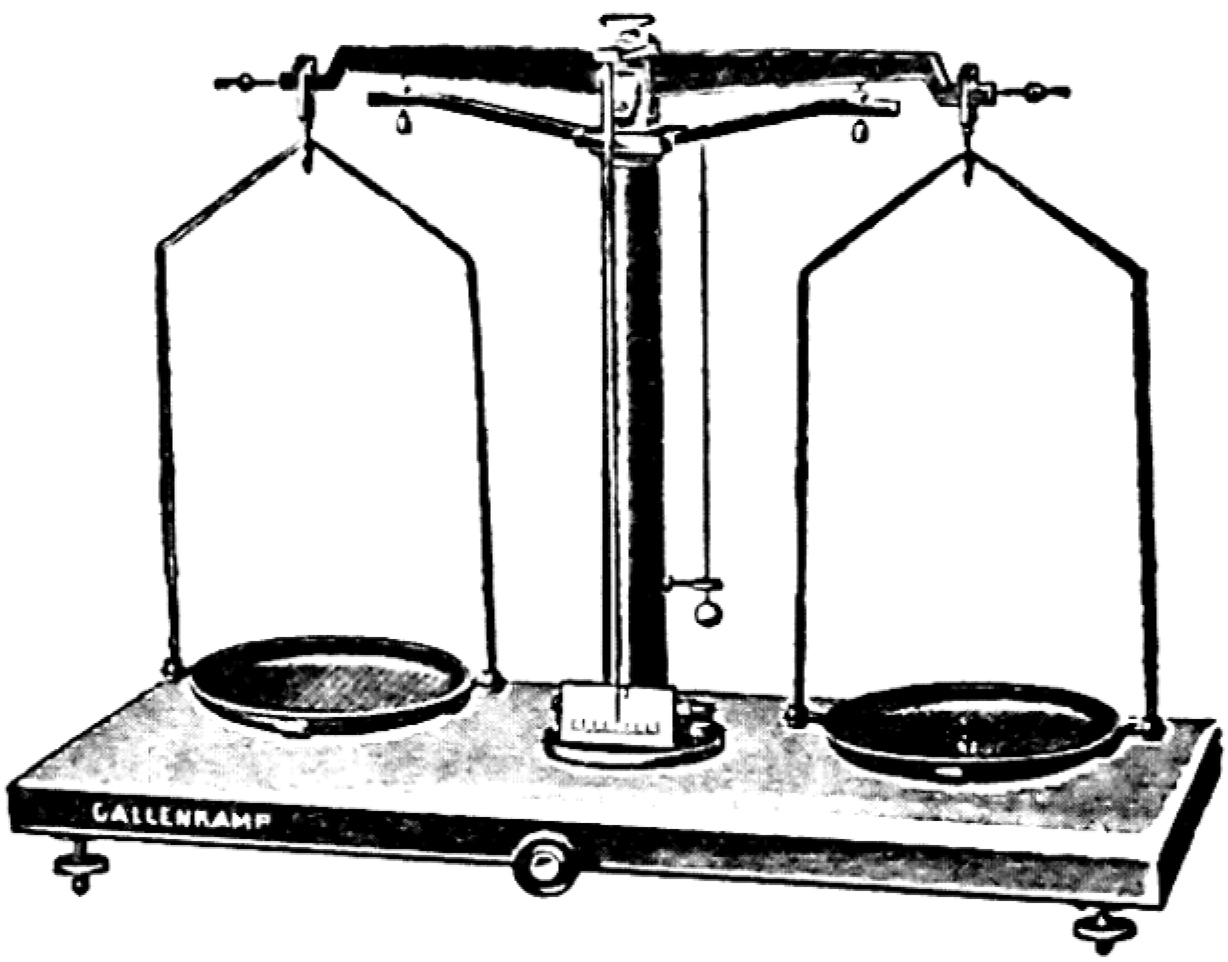


FIG. 2.—BALANCE.

(usually) at its centre, to which two equal scale-pans are attached. The arm carries a pointer (Fig. 2) which moves over a short scale near the base of the instrument. When the balance is released, by raising the arm off the stops, it should swing gently to and fro so that the pointer makes equal excursions on either side of the zero of the scale, opposite which it ultimately comes to rest. If its excursions are to one side the leverages of the two sides are not balanced. This may be adjusted by lowering the arm on to the stops

and screwing a nut along at one end, thus increasing or decreasing the leverage of its mass until the balance swings equally. Placing the object to be weighed on one scale-pan, one starts adding weights to the other until (within the limitations of the particular box of weights) the addition of the smallest ' weight ' is sufficient to topple the balance over to the other side.

Just as in our measurement of length, we have then circumscribed the mass of the object between two values, the sum of the weights before and after the increment of the smallest one available (either 0·1 gm. or 0·01 gm., according to the class of box). Fractions beyond this are determined by a rider, a little mass which bestrides the arm and can be moved along a scale inscribed on it until equality is attained. The rider takes the place of the vernier in length measurements.

The measurement of time is carried out by the principle (to be discussed more fully under Sound) by which the time of oscillation of a body set in motion under restoring forces depends on its length. Thus in the simple pendulum, consisting of a bob on the end of a thin string, set in oscillation through a small arc, the time of each swing depends on the distance of the point of support from the centre of the bob. If the thread is 24·8 cm. long, the pendulum beats seconds—at London; its time of swing varies with the acceleration due to gravity, and therefore with the position on the earth's surface.

The measurement of temperature will be more fully discussed in the section on Heat, but it may be said now that it is usually measured on a *thermometer* by the expansion of a body subjected to the temperature, for example, of a liquid column whose expansion causes it to move along a graduated glass tube.

(The reader will observe how important is the estimation of a length in a physical laboratory, since the physicist prefers to make all his measurements in that way. Thus, the balance has a pointer moving over a scale, the time-marker has a pendulum cut off to a measured length, and the thermometer has a column of liquid moving along a

scale. This is partly due to convenience—one cannot conceive of a second or of a degree of temperature being kept, as such, in a laboratory cupboard—and partly because the physicist places more reliance on his sight than on his other senses in estimating mass, time, and temperature. The same principle applies in other branches of physics—most electrical instruments have pointers moving over scales. Even so, as we have already stated, the use of sight is restricted, whenever possible, to observe when two marks coincide, and not to compare distances as such.)

The Estimation of Accuracy

In expressing the results of an experiment their accuracy should always be estimated. Just as the speed of a fleet is limited to that of the slowest ship, so the accuracy of the final result of an experiment cannot be greater than any single measurement involved in it. Beginners in practical measurement tend to quote more figures in a result than can be justified by this rule, often because they have used logarithms in the calculation and the tables give logarithms to four or five figures.

To take a simple example; if the volume of a box is being measured and the observer reckons that he can only measure a length correctly to the nearest millimetre, the resulting volume cannot be true even to a cubic millimetre, and any figures derived in the multiplication which express fractions of a cubic millimetre must be rejected when writing down the final result.

Another instance from outside the laboratory: In a competition to compare the 'all-round' capabilities of athletes, their best times over, say, 100 yds., 440 yds., and a mile may be taken and the average speed worked out; if the two best competitors have averages quoted as 6·453 m.p.h. and 6·452 m.p.h., this does not mean that the first has won the competition unless it can be proved that the times for a 100 yds. can be accurately estimated to half a second, since ·001 mile per hour makes a difference of this order in the times of the two competitors.

Graphical Representation

It is often helpful to express the results of an experiment in graphical form. If there are two dependent variables—for instance, if the temperature of a furnace at a factory is under observation from time to time—a piece of squared paper will be taken, and one of these variables—let us say temperature—will be plotted vertically as *ordinate*, and the other, time, horizontally as *abscissa*. Some thought should

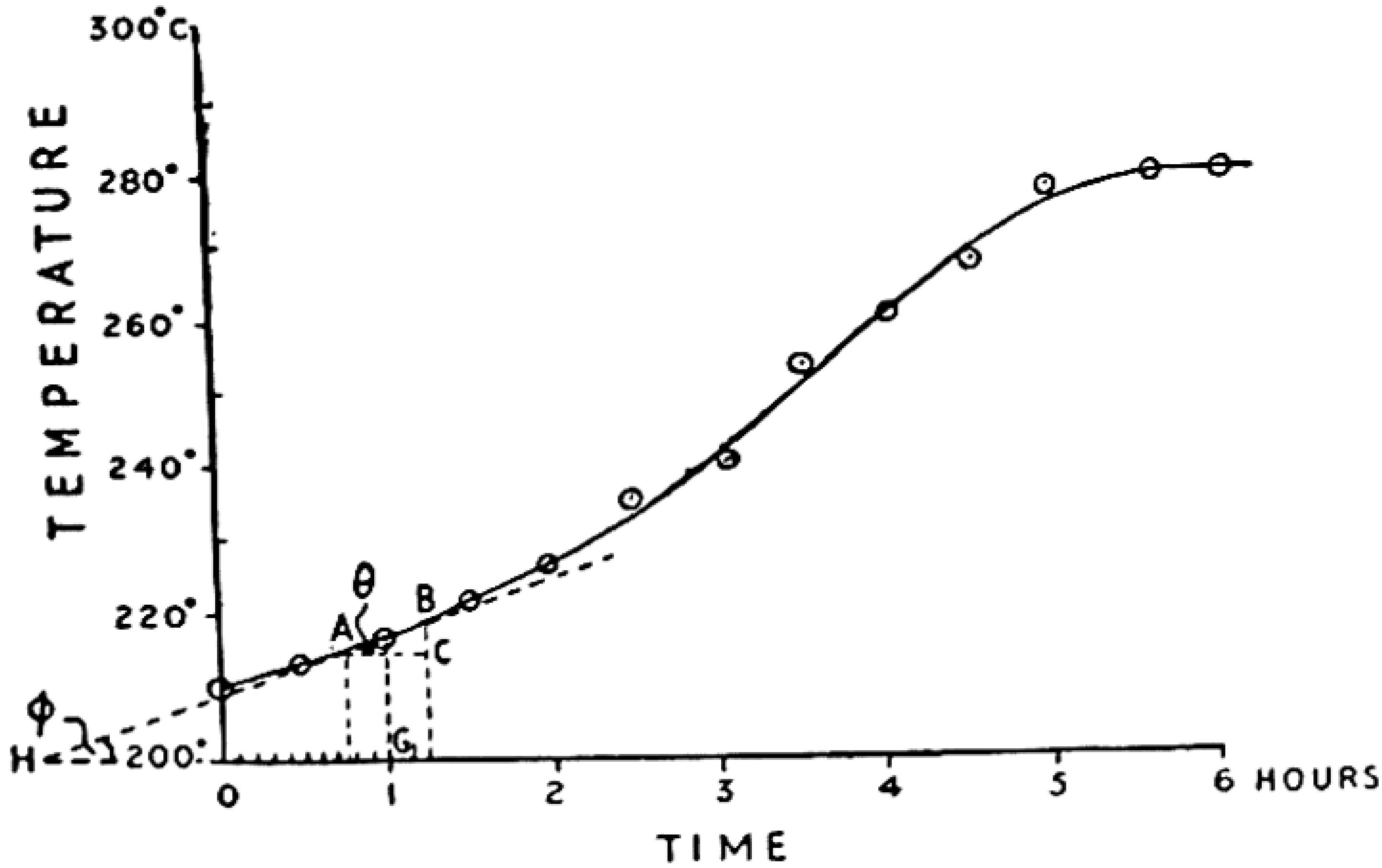


FIG. 3.—RATE OF CHANGE OF A PHYSICAL QUANTITY.

be given to the scales before the points are plotted. The curve should fill the allotted space and not be squeezed into one corner. This means that if the lowest temperature recorded is 210° C. and the highest 280° C., the former should be near the bottom and the latter near the top of the vertical scale; at the same time the scale adopted should be convenient, in the sense that the number of squares allotted to each unit of the variable should be a whole number, or if one square has to represent several units, it should be made equal to a small multiple or submultiple of ten. In the case

of the furnace record, it would be convenient to start with 200° C. at the bottom and let each small square side equal 10° C. If the observations covered 6 hours, say, one would allot one small square side horizontally to every 6 minutes, so that 6 hours = 6 large squares. Points of observation are marked by circles or crosses, each temperature over the corresponding time, and a smooth curve drawn through them (Fig. 3). Notice that, in default of information to the contrary, we do not join the points by straight lines, to make a zig-zag trace. The latter would imply that the temperature rise or fall changed suddenly just at the instant when we were lucky enough to make our observations, which is unlikely.

If there are three variables, it is possible to exhibit their relationships by plotting two of them in a family of curves, to which a 'label' is fixed. If, for instance, our experiment were repeated with different thicknesses of lagging round the furnace, the latter curves could be drawn on the same piece of paper, and the appropriate thickness of lagging marked against each curve.

Rate of Change, Average and Instantaneous

Having plotted a curve of this type to show how a certain factor varies as time goes on, it is often desired to deduce from it the *rate* at which the factor is varying. This may be done roughly or more accurately, according as the period over which we want to know the rate is long or short. Thus we observe that the temperature was 213° C. at 1 hour (from the start) and 222° C. at $1\frac{3}{4}$ hours, and accordingly we might say that the rate of rise of temperature at that time was about 12° C. in an hour. This, however, is only an average rate, and does not represent the true state of affairs at every minute in the period. If we are asked for the 'rate of change' at 1 hour from the start, we can get a better answer by the difference of the temperatures at a quarter of an hour before and after the hour, dividing by the time interval (i.e., 30 minutes). The two quantities involved are shown as *BC* and *AC* on the graph, and, together with the line joining the two points on the curve

(AB), form a triangle having a right angle at C and an acute angle (θ) at A.

Thus the rate of change of temperature over the period of time $EF(= AC)$ is given by:

$$\frac{\text{Number of temperature units represented by } BC}{\text{Number of time units represented by } AC}$$

The result is still only an average, but over the shorter period of 30 minutes. Now imagine this period of time to get smaller and smaller by taking the points F and E closer to the hour. This makes the triangle ABC become smaller and smaller as the points A and B approach each other along the curve. Eventually they will coincide at the point D on the curve, vertically over G, and the line AB, which was a secant to the curve, becomes the geometrical tangent making an angle ϕ at H with the base line of the graph.

The ratio BC/AC, which was equal to $\tan\theta$, now becomes DG/HG, which is equal to $\tan\phi$. This quantity is also known as the *slope of the curve* at D, and represents the *instantaneous rate of change* at this point. It is important to notice that the slope of the curve in this sense (namely, the tangent of the angle which the geometrical tangent to the curve makes with the base line) is not equal to the instantaneous rate of change, unless the units have been chosen so that the same number of squares on each axis equals the same number of units of each variable. It is not advisable, in calculating the rate of change, to draw the line DG and look up $\tan\phi$ in mathematical tables; one should measure DG and convert the number of squares it covers into units of ordinate and do the same for HG in terms of units of abscissa, then divide. Thus instantaneous rate of change of temperature at 1 hour

$$= \frac{\text{Number of temperature units represented by } DG}{\text{Number of time units represented by } HG} = \frac{13^\circ \text{ C.}}{1\frac{2}{3} \text{ hr.}}$$

or 8° C. per hour.

Trigonometry

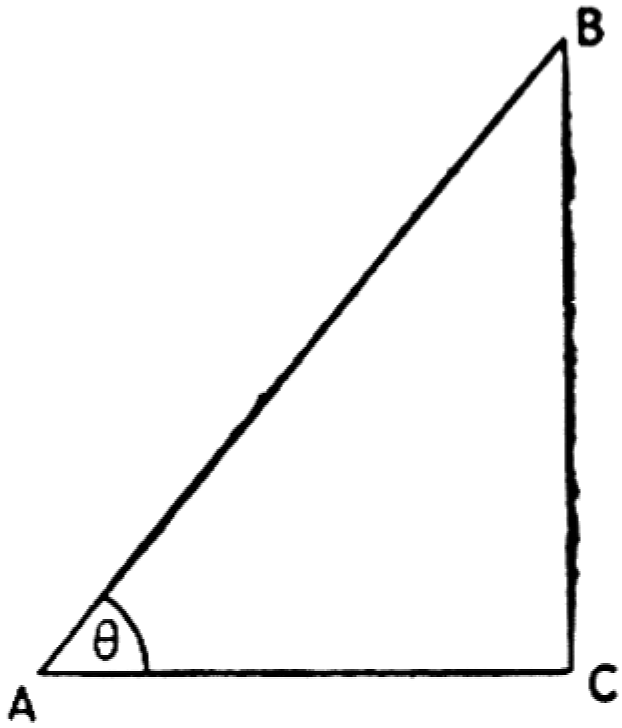
In a right-angled triangle such as *ABC* the following names are ascribed to the ratios of the sides in reference to the angle :

$$\text{sine } \theta \text{ (or } \sin\theta) = \frac{BC}{AB} = \frac{\text{height}}{\text{hypotenuse}}$$

$$\text{cosine } \theta \text{ (or } \cos\theta) = \frac{AC}{AB} = \frac{\text{base}}{\text{hypotenuse}}$$

$$\text{tangent } \theta \text{ (or } \tan\theta) = \frac{BC}{AC} = \frac{\text{height}}{\text{base}}$$

There are others, but these suffice for the present text.



This book belongs to
Rila. Hussain. P. U. C.

HEAT SECTION

CHAPTER II

EXPANSION BY HEAT

Introduction

How did man first appreciate the presence of heat? For long periods of time the sun would be the only source which was obvious to his primitive brain, unless he was living in a volcanic region. Even there it is unlikely that he detected any common factor in the fiery eruptions and the sun's rays. Of the ability of the sea to act as a temporary storehouse of some of the sun's heat and exert a beneficent effect on climate he may have had slight knowledge.

The first step in man's direct utilization of heat for his own purposes was taken when he discovered how to make fire. The manner of its discovery will probably never be known. News of this novelty would spread to some extent, but many isolated communities must have re-discovered it for themselves. Slowly some of its uses became part of everyday life. It could provide warmth when the sun hid its face, and Charles Lamb gives a delightful account of the first roasting of pork, though he would never have claimed it as an unimpeachable source of information. Later fire took its place, often associated with the sun, in religious life. Sun-worship, together with the necessity for determining the most suitable times for seed-planting, encouraged a study of the motion of the sun in the heavens and of its waxing and waning with the seasons. At Stonehenge there exists a sighting line marked with stones which indicated the most northerly position of sunrise—at the time of the summer solstice. There is evidence that this study has been undertaken by peoples separated as far as the Eskimos of Greenland, the natives of the South Sea islands, and the Incas of Peru.

The use of heat in the extraction and working of metals

began in very early times, and was probably first applied to copper, gold, and silver. The supremely important rôle played by metals in the development of man's control over nature needs little emphasis. Efforts to find improved methods of refining metals stimulated the study of chemistry. The metals themselves have had a revolutionary effect on almost every sphere of man's activity, even agriculture being within the scope of their application. It is probably in the development of engines to derive power from the heat produced by the burning of fuels that most remarkable advances have been made. Most people have heard of the heat engine, attributed to Hero of Alexandria (about 100 B.C.), which consisted of a vessel, pivoted at top and bottom on vertical axles, and provided with jets. On filling the vessel with water and placing it over a fire, steam issued from the jets and caused the vessel to recoil and revolve in the opposite direction to that of the motion of the steam. From this small beginning, which had no immediate practical use, there followed, after a barren interval of more than 2000 years, the engines of Savery, Newcomen, and Watt, culminating in the steam turbine of Parsons, which provides the power needed to supply electricity in our large cities and to drive giant liners.

This brief survey is sufficient to demonstrate that the study, control, and use of heat have produced most formative influences in the slow evolution by which modern modes of living have been produced.

Sources of Heat

The principal sources of heat are as follows:

(*a*) That reaching the earth from outer space (almost entirely from the *sun*);

(*b*) That obtained when *fuels* are burnt. The principal fuels are coal and the gases derived from it, wood, oil, and natural gas.

(*c*) That obtained when an *electric current* passes through a resistance. The energy needed to produce

the current may have been derived from coal, oil, or water power.

(*d*) That produced in some *chemical reactions*. For example, when cold, strong sulphuric acid is added to cold water, the resulting liquid will be warm. The energy in *food* is also partially changed into heat when it undergoes chemical processes inside the body.

(*e*) Some of the heat from the interior of the earth escapes in large quantities in *volcanic* regions. Thus, in Iceland natural hot-water springs are used to supply central-heating systems. Radioactive substances, such as radium, continuously emit heat, and may have a considerable effect in maintaining the temperature of the earth.

(*f*) Wherever there is *friction*, heat is produced. Primitive tribes still produce fire by rapidly rotating a stick in a cavity in a piece of wood. In modern life it is well known that if the lubricating system of an aircraft engine fails, the bearings become hot and will finally ' bend ' due to the heat produced by friction in the bearing surfaces.

Heat and Temperature

The term ' temperature ' is known to most people, but its precise meaning is less familiar. What is it that causes water to flow from one point to another? It is the difference of level between the points. Similarly, heat will flow from one body to another only if there is a difference of temperature between the bodies. *If heat flows from a body* A *to a body* B, *then* A *is said to be at a higher temperature than* B.

Temperature is usually measured by the expansion either of a liquid column in a glass tube, or (for higher temperatures) of a solid rod. In the common liquid-in-glass thermometer the scale is inscribed by reference to two *fixed points*, which are usually:

Lower Fixed Point; the temperature at which ice melts.

Upper Fixed Point; the temperature at which pure water boils under normal atmospheric pressure.

The liquid is contained in a bulb, holding a few c.c., to which a stem is sealed. To mark the fixed points, the bulb is placed first in a mixture of melting ice and water, and the level at which the liquid (usually mercury) settles is marked on the stem. Next the bulb and as much of the stem as possible are immersed in the steam arising from boiling water, and the upper fixed point marked where the mercury stands in the stem.

The space between the fixed points can be marked out on two different scales:—

The Centigrade Scale. The fixed points are 0° C. and 100° C., respectively, so that there are 100° between them.

The Fahrenheit Scale. The fixed points are 32° F. and 212° F., respectively, so that there are 180° between them.

Thermometers are made having a more limited range for special purposes—for example, for calorimetric work (p. 50) or for medical use.

Heat is a form of energy, and the higher the temperature of a body, the more heat it contains. Nevertheless, heat and temperature are distinct and different. For example, if a small, white-hot metal ball could be placed in the water contained in the boilers of an Atlantic liner, heat would pass from the ball to the water. This is because the ball is at a higher temperature than the water. Nevertheless, the large quantity of boiling water would contain much more heat than the ball.

The Effects of Heat: Expansion of Solids

Suppose a piece of steel is steadily heated from room temperature. At first the heat has no effect which can be seen, but soon the steel will become dull red. This occurs at a temperature of about 500° Centigrade. At about 600° C. the steel will be bright red, and when it reaches

1200° C. it will be white hot. This is one obvious effect of heat on a substance.

All solids expand when heated. Thus when copper is heated from 0° to 1° C. it increases in length by ·000017 of its length at 0° C. For glass, the average fractional increase is ·0000085, while for the alloy 'invar' it is so small as to be negligible.

It is well known that a gap is left between successive rails in railway lines. A section of rail 60 ft. long at 0° C. (freezing point) would be $\frac{1}{3}$ in. longer on a hot summer day when the temperature was 40° C. Trials of welded rails in considerable lengths on London's underground railway have been successful because the variation between day and night temperatures and between summer and winter temperatures is small there. The fact that tram-rails can be used without leaving gaps is interesting. This is because the temperature of the earth even just a few inches below the surface varies much less than that of the air.

In fixing overhead telegraph wires allowance must be made for expansion and contraction. Steam-pipes usually incorporate a U-shaped section. The mouth of the U is made smaller or larger as the pipes on either side of it expand or contract. In the great Forth Bridge, rollers admit of a total movement of 4 ft. in the girders between their minimum and maximum length. It is sometimes necessary to fix a cylindrical tube securely inside another, as for example in fitting a liner to a cylinder in an internal-combustion engine. To do this the external diameter of the liner is made slightly larger than the internal diameter of the cylinder, at ordinary temperatures. The liner is then cooled in liquid air (which boils at about −185° C.), and so shrinks. The amount of shrinkage in the external diameter has been calculated, so that the liner can now just slide into the cylinder. When normal temperature is reached again, the liner has expanded and is securely fixed in position. The flanged tyres are fitted to locomotive wheels by the reverse process of heating the tyre and fitting the wheel inside it while the tyre is still hot.

'Invar', an iron alloy containing 36 per cent. nickel,

expands so little that for small temperature changes its dimensions can be considered constant. For this reason it is sometimes used as a material for clock pendulums.

The period of vibration of a watch is controlled by the balance wheel and the strength of the hair spring. The rest of the watch mechanism is fundamentally a device for counting the number of vibrations of the balance wheel. If the period of vibration increases, the watch loses. The effect of a rise in temperature is to increase the size of the wheel and to weaken the hair spring. The result of each of these changes is to make the watch lose, the latter effect

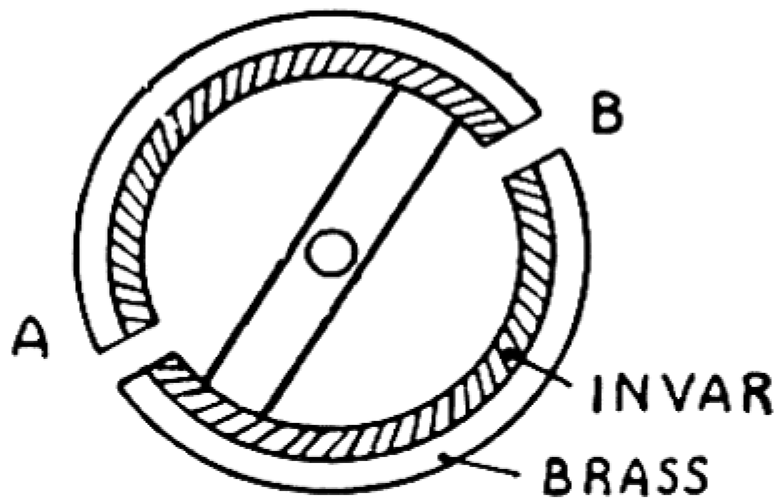


FIG. 4.—COMPENSATED BALANCE WHEEL.

being the larger. The weakening of the spring cannot be overcome, but the balance wheel can not only be prevented from increasing in diameter as it becomes warmer, it can even be made to become smaller. The decrease in diameter can be made of such a magnitude as to compensate for the effect of the weakening of the spring, thus making the rate of the watch independent of temperature changes. This is done by making the balance wheel from two metal strips riveted together, one being of brass and the other of invar, the latter being arranged on the inside of the wheel. The wheel is in two parts (Fig. 4). As its temperature rises, the invar remains almost unchanged in length, but the brass expands. This causes the free ends of the wheel segments *A*, *B* to move inwards, producing the result described above.

It will make our ideas about the expansion of solids more precise if we make some actual measurements. Take a

brass or steel tube (A) about 50 cm. long, and solder side tubes (B, C) near each end (Fig. 5). Support the tube on a wooden base, and arrange a block of wood (D) at one end to act as a stop. At the other end fix a micrometer screw gauge (E). Let cold water flow through the tube for several minutes and read the temperature of the outflowing water with a thermometer. Suppose this is 12° C. While the water is flowing, measure the length of the tube to the nearest millimetre. Suppose it is 50·4 cm. Turn the micrometer

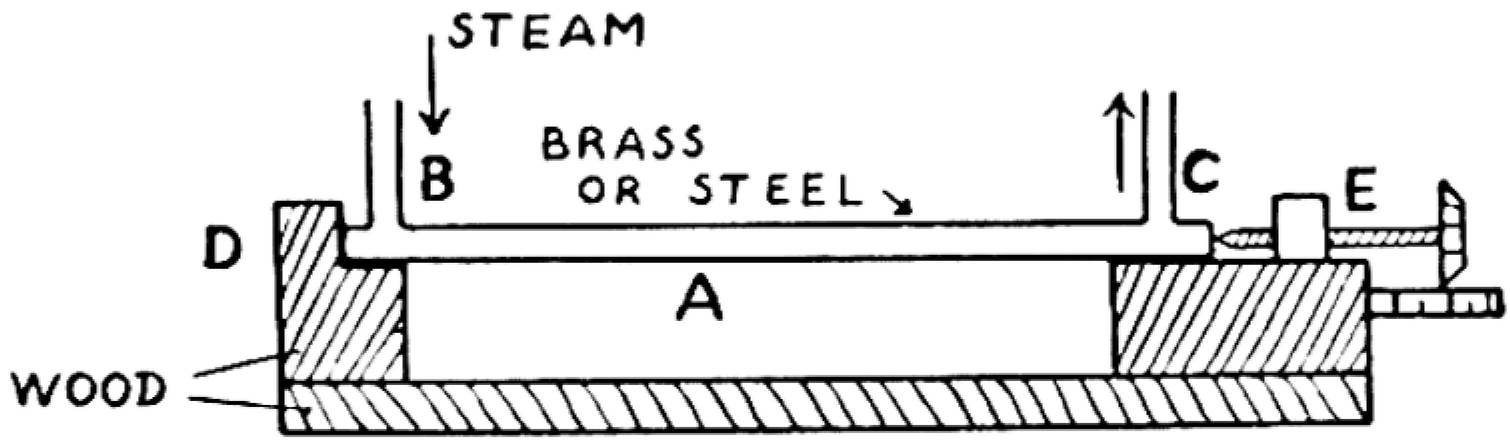


FIG. 5.—EXPANSION OF A ROD.

until the end of the screw just touches the end of the brass tube. A sheet of white paper placed behind the screw will help in determining the position where the screw first makes contact with the tube. Note the reading on the micrometer, and then unscrew it so that there is again a gap between it and the tube. Suppose it reads 2·74 mm. Cut off the flow of water and in its place pass steam through the pipe. The steam can be generated by heating water in a metal can over a gas flame and led to the expansion tube by a length of rubber tubing. Allow the steam to pass for at least 10 minutes.

It may then be assumed that the temperature of the tube is 100° C. within $\pm\frac{1}{2}$° C. (If greater accuracy is desired the temperature of the steam when entering and leaving the tube should be measured and the mean of the two results taken.) The micrometer screw should again be turned till it just makes contact with the end of the brass tube, and a new reading taken. Suppose it is 3·58 mm.

Then a brass tube, 50·4 cm. long at 12° C., has expanded

$3{\cdot}58 - 2{\cdot}74 = {\cdot}84$ mm. $= {\cdot}084$ cm. on being heated to 100° C. Hence if it had been heated only 1° C. (from 12° to 13° C.) it would have expanded by $\frac{{\cdot}084}{100-12} = {\cdot}00096$ cm. (to 5 decimal places). If the tube had been only 1 cm long, the expansion would have been only $\frac{{\cdot}00096}{50{\cdot}4} =$ ${\cdot}00001905$ cm. Our experimental results were not taken with sufficiently precise apparatus or care to justify giving the answer to 8 figures, so we will approximate this answer to ·000019 cm. The answer means that if we heat a piece of brass 1 cm. long through 1° C. it will expand by ·000019 cm. *The amount by which a substance of unit length* (i.e., 1 cm., 1 m., 1 ft., or any other unit) *at* 0° *C. expands when it is heated to* 1° *C. is called* the coefficient of linear expansion. The value of ·000019 cm. we obtained was for the expansion of 1 cm. of brass at 12° C., since the length 50·4 cm. at 12° C. would contract by $50{\cdot}4 \times {\cdot}000019 \times 12 = {\cdot}0114912$ cm. if cooled to 0° C. assuming that the expansion is uniform between 12° and 0° C. So we should really divide the expansion of ·00096 by $(50{\cdot}4 - {\cdot}0114912)$ instead of by 50·4. However, it will be clear that there is little difference between these latter quantities. Hence for most practical purposes the first method we used for calculating the coefficient of linear expansion can be considered sufficiently accurate.

If one takes a square piece of steel which has sides 1 cm. long at 0° C., then at 1° C. each side will be $(1 + {\cdot}000012)$ cm. long. So the area of the steel will now be $(1 + {\cdot}000012)/(1 + {\cdot}000012) = 1 + 2 \times {\cdot}000012 + {\cdot}000012^2$ sq. cm. It is clear that ${\cdot}000012^2$ is a very small quantity, so for practical purposes the *area has increased* by $2 \times {\cdot}000012$ sq. cm., that is, *by twice the coefficient of linear expansion*, on being heated through unit rise of temperature. This increase in area produced when unit area is heated through unit rise of temperature is called the coefficient of areal (or superficial) expansion. In Fig. 6 the shaded portion shows the total increase in area in the case

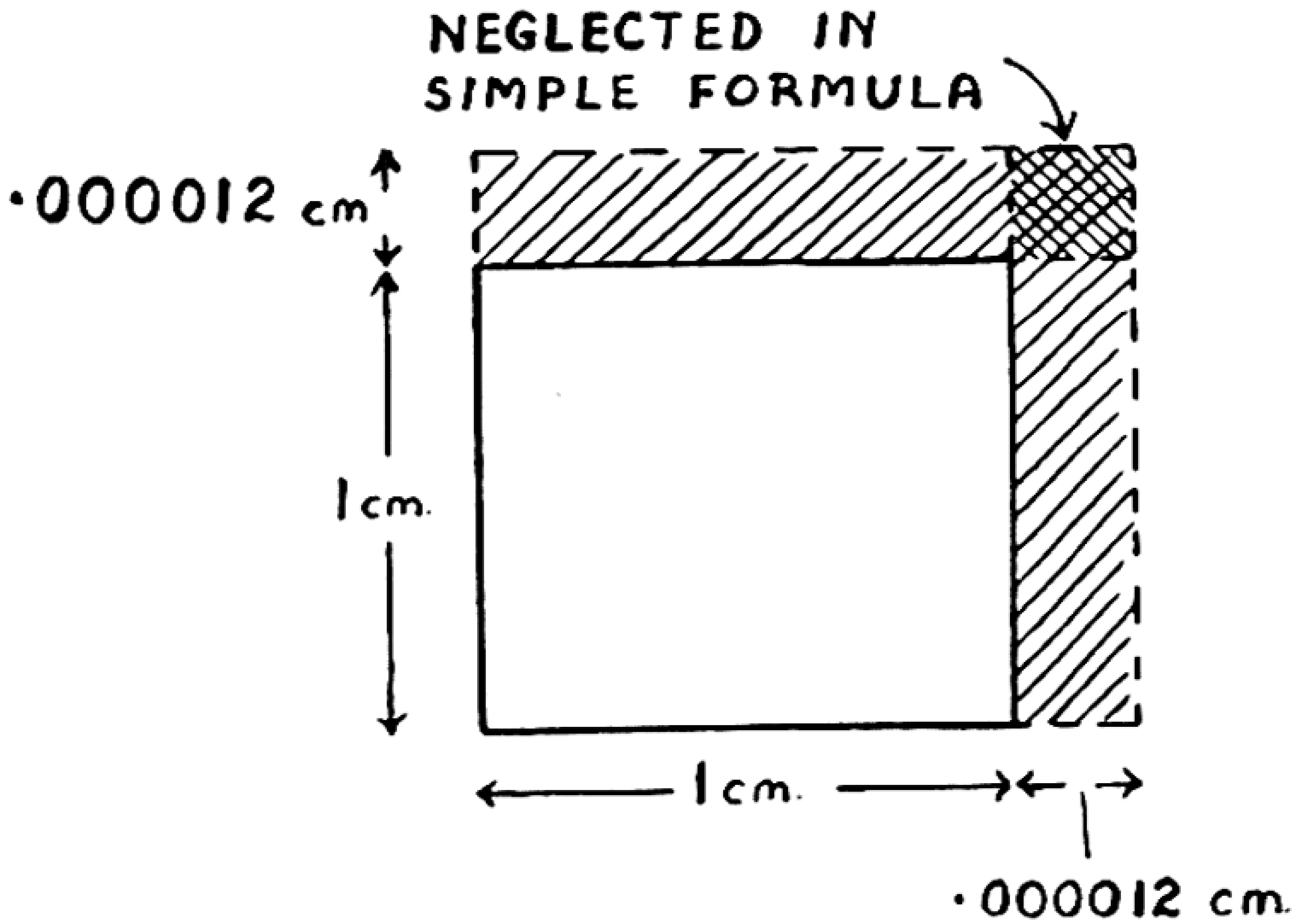


FIG. 6.—EXPANSION OF A PLATE.

of steel. The double-shaded portion is neglected in the simple formula. It will be easily seen that the remainder

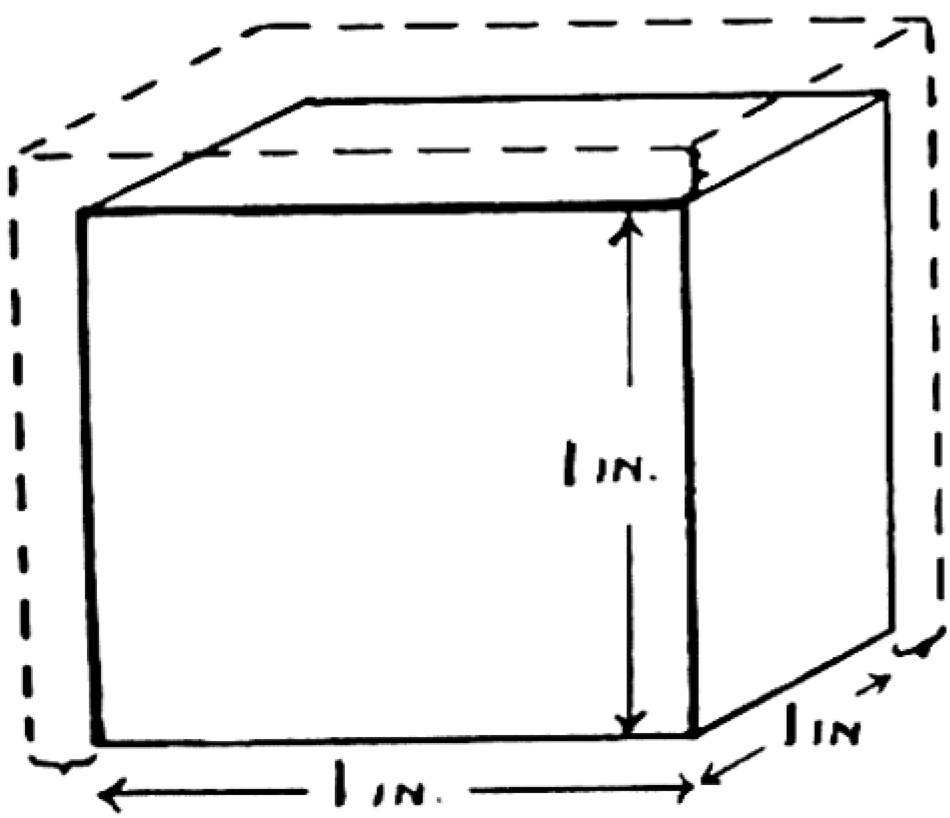


FIG. 7.—EXPANSION OF A CUBE.

of the increase in area amounts to $2 \times \cdot 000012$ sq. cm. Though steel has been taken as an example, the above definition (though not the numerical result) is true for any solid, and for any unit of area (e.g., the sq. inch, the sq. foot, or the sq. metre).

In terms of volume we commence with a zinc cube of, say, 1 in. side at 0° C. (Fig. 7). For zinc the coefficient of linear expansion is ·000029 per ° C. Hence if the cube is heated to 1° C., each side will become $(1 + \cdot 000029)$ in., and the volume will increase from 1 cu. in. to $(1 + \cdot 000029)^3 = 1 + 3 \times \cdot 000029 + 3 \times (\cdot 000029)^2 + (\cdot 000029)^3$ cu. in. The terms involving $(\cdot 000029)^2$ and $(\cdot 000029)^3$ can be neglected, since they are extremely small. Hence on heating 1 cubic inch of zinc from 0° C. to 1° C. it has increased in volume by $3 \times \cdot 000029$ cu. in. This *increase in volume produced in unit volume by a unit rise of temperature is called the* coefficient of cubical (or volume) expansion, and it will be seen that *its value is three times the coefficient of linear expansion.* In Fig. 7 the expansion of each of the three 1-in. sides is represented (not to scale) by the bracketed length.

Expansion of Liquids

One of the simplest demonstrations that liquids expand when heated is to be found in a simple *thermometer.* The usual liquids contained in such a thermometer are mercury or alcohol. In either case, when the thermometer is placed near a source of heat the glass of the bulb and the stem expands, but the liquid inside does so much more. Hence the liquid rises up the stem and indicates an increase of temperature.

Mercury expands very regularly with rise of temperature. The amount by which 1 cubic centimetre of mercury expands when heated from 0° to 1° C. is almost identical with the expansion produced when the same volume is heated from say 40° to 41° C. or 60° to 61° C. That is one of the reasons why mercury is so much used as a *thermometric liquid.* The expansion of alcohol is less regular, and that of water is markedly irregular. As shown in Fig.

8, from the melting point of ice (0° C.) up to 4° C. water becomes denser—that is, it contracts—when heated. Above 4° C. it acts more normally and expands when heated. An application of this fact in nature is of great importance to water life. Without it any aquatic creatures

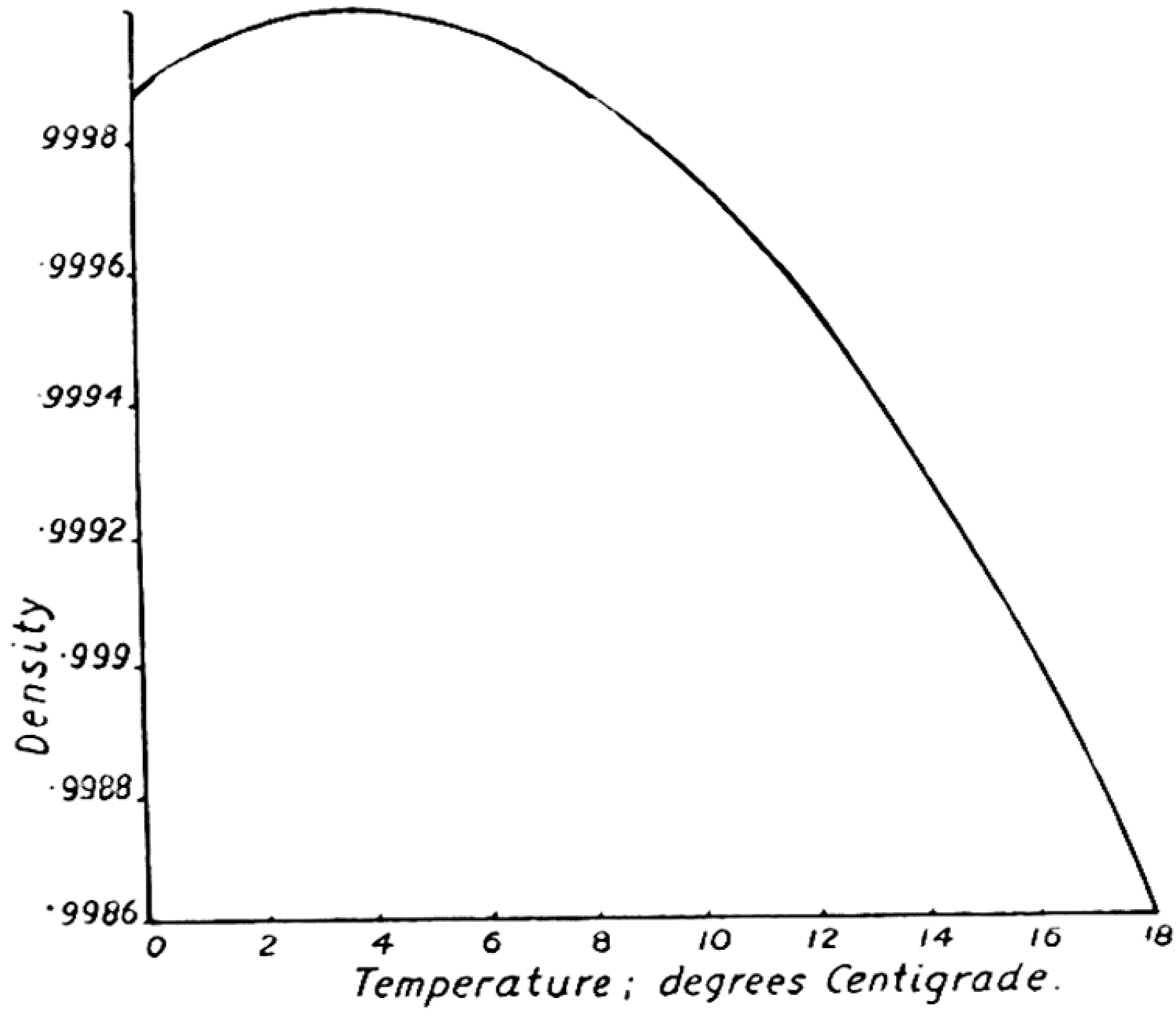


FIG. 8.—DENSITY OF WATER AND ITS VARIATION WITH TEMPERATURE.

which could not survive being frozen would be killed each winter. For consider a pond, initially at say 10° C., which is gradually cooled on a frosty night. The wind passing over its surface cools the upper layers of the water, and makes it heavier or denser. The cooled water therefore falls to the bottom of the pond and displaces some of the lighter, warmer water to the surface, where it, too, is cooled. This steady circulation of the cooled water continues until

the temperature of the pond is 4° C., simply because if water above 4° C. is cooled it becomes heavier. At 4° C., however, as Fig. 8 shows, a change takes place. Below that temperature, as water is cooled, it becomes lighter, and so remains on the surface. Since it is no longer replaced by warmer water from below, the surface layer thereafter cools rapidly and turns into ice. It takes a prolonged spell of cold weather to cause a deep pond to freeze to the bottom. (The reason for this will be clearer when we have discussed the ways in which heat travels.) Hence, though the surface of the pond is frozen, water is left below for the fish.

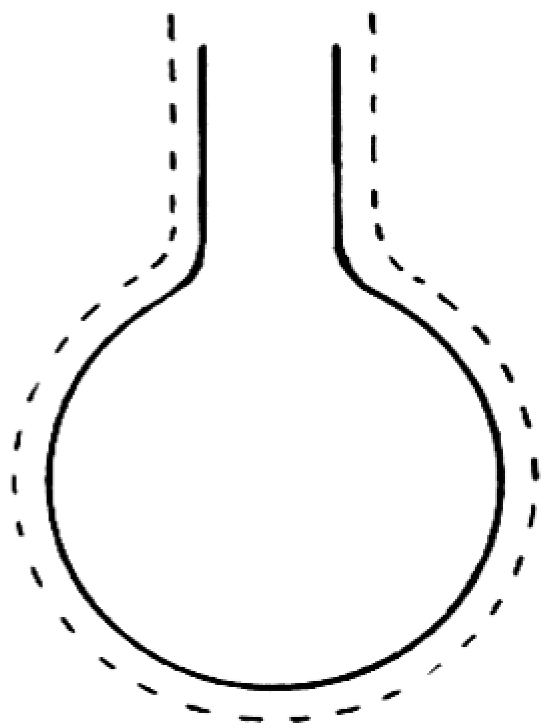


FIG. 9.—APPARENT EXPANSION.

Consider a thin-walled glass vessel almost filled with a liquid. If the vessel is heated from outside, the vessel will first expand, and momentarily the level of the liquid will fall. Then the heat will reach the liquid, the expansion of the liquid will greatly exceed that of the vessel, and the level of the liquid will rise. The amount of the expansion of the vessel will be the same as it would have been if the vessel had been of solid glass (Fig. 9). This expansion will actually have no effect on the amount of expansion of the water, but if no allowance is made for the increase in size of the vessel, the water will appear to have expanded less than it should have done. For this reason the expansion of any liquid or gas, in which the effect of the expansion of the containing vessel is disregarded, is known as the **apparent expansion**, to distinguish it from the **real expansion**.

The volume of water in the vessel, when full, will obviously be the same as that of the interior of the vessel. The temperature of the water will differ only slightly, if at all, from that of the vessel. Now, the expansion of any volume of material is given by the product of its initial

volume, the rise in temperature, and its coefficient of cubical expansion. Hence if the vessel and the water (representing two materials) have the same initial volume and experience the same rise in temperature, the amount by which they expand will be proportional to their real coefficients of cubical expansion. So when, from an experiment, we find the apparent coefficient of expansion of a liquid in a vessel, *its real coefficient of expansion can be found by adding to the apparent coefficient the real coefficient of expansion of the material of the vessel.* Expressed as an equation this becomes:

Real coefficient of expansion of liquid in a vessel =
Apparent coefficient of liquid in vessel +
Real coefficient of material of vessel.

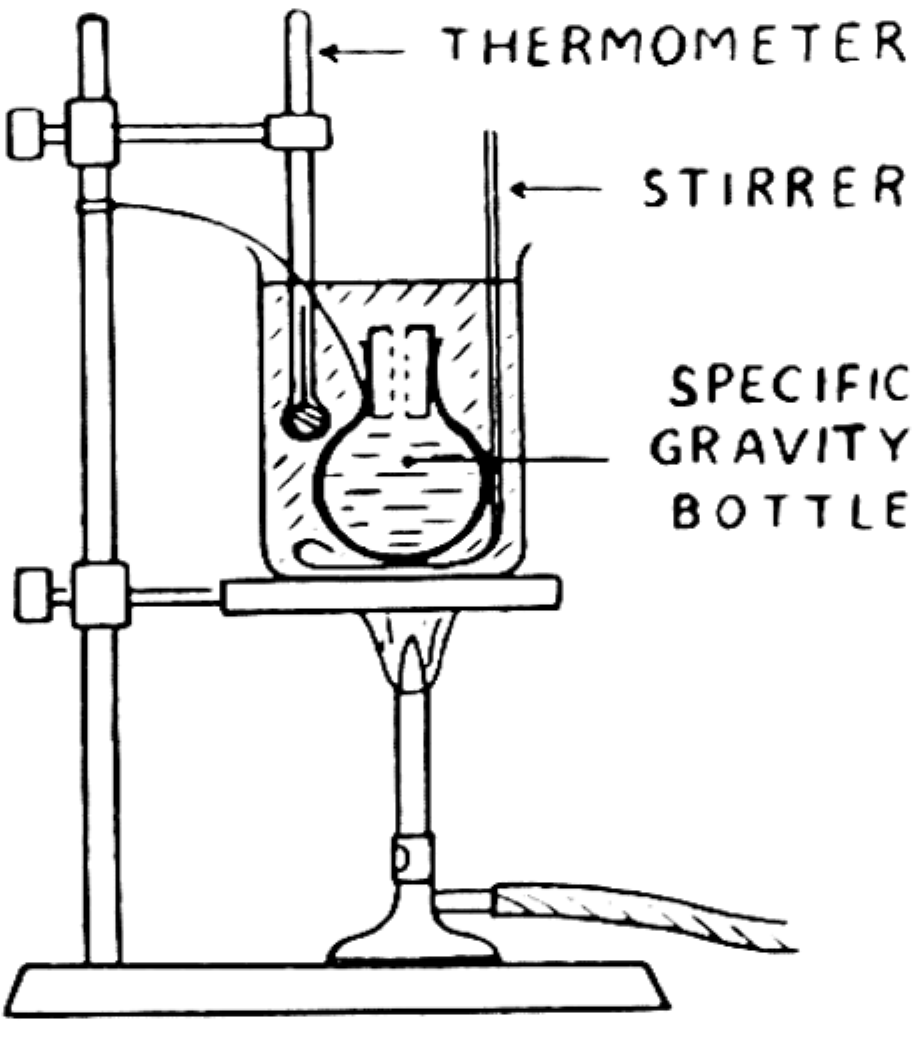


FIG. 10.—EXPANSION OF A LIQUID.

To Find the Apparent Coefficient of Expansion of Water

A specific-gravity bottle is the most suitable vessel for this experiment. It is a bottle (Fig. 10) having a ground-glass stopper with a hole of small bore through its length.

If the bottle is filled with water, the stopper inserted gently in place, and the bottle wiped dry, it contains a definite volume of water at a given temperature. The bottle will be full to the top of the bore in the stopper, and this volume of liquid can be exactly reproduced at any subsequent time, if the temperature is the same. If a bottle of this type is not available, a small glass bottle having a ground-glass stopper may be substituted. With carborundum powder grind a channel down the length of the stopper to act as an outlet for liquid overflowing from the bottle. Into some cold water place some ice, and allow it to cool the water down to 0° C. Weigh the bottle and stopper, together with a watch-glass. Measure the temperature of the ice-cold water with a thermometer. Suppose the result is 0° C. The bottle should be filled with this water and the stopper gently replaced. The bottle is then dried, placed on the watch-glass, and the whole weighed. The temperature of the water will slowly rise, since it is no longer in contact with the ice. It will therefore expand and overflow from the bottle, during the weighing. The purpose of the watch-glass is to collect this overflow.

Now place the bottle in a beaker of water, which can be heated. The bottle should be kept in a vertical position by means of thread round its neck, fastened to some support external to the beaker (Fig. 10). The retort stand shown is useful, but not essential to the experiment. Let the temperature rise slowly to about 50° C., and stir the water in the beaker thoroughly. As the stated temperature is approached, reduce the input of heat and endeavour to keep the temperature constant for a minute or two. The maximum temperature reached should be observed, and then the bottle removed from the beaker, dried externally, allowed to cool, and re-weighed, together with the watch-glass. The results of an actual experiment were as follows :

(3) Weight of bottle full of water at 50° C. + watch-glass . . . = 72·36 gm.
(2) Weight of bottle full of water at 0° C. + watch-glass . . . = 72·64 gm.

(1) Weight of empty bottle + watch-glass . . . . . = 47·26 gm.

The numbers at the left give the order of weighing. Taking weighing 1 from weighing 2, and weighing 1 from weighing 3 we get:

Weight of water filling bottle at 0° C. = 25·38 gm.
Weight of water filling bottle at 50° C. = 25·10 gm.
∴ Weight of water expelled from bottle in heating it from 0° to 50° C. = ·28 gm.

The 25·1 gm. of water which filled the bottle at 50° C. would not fill it at 0° C. If, however, we reheated this mass

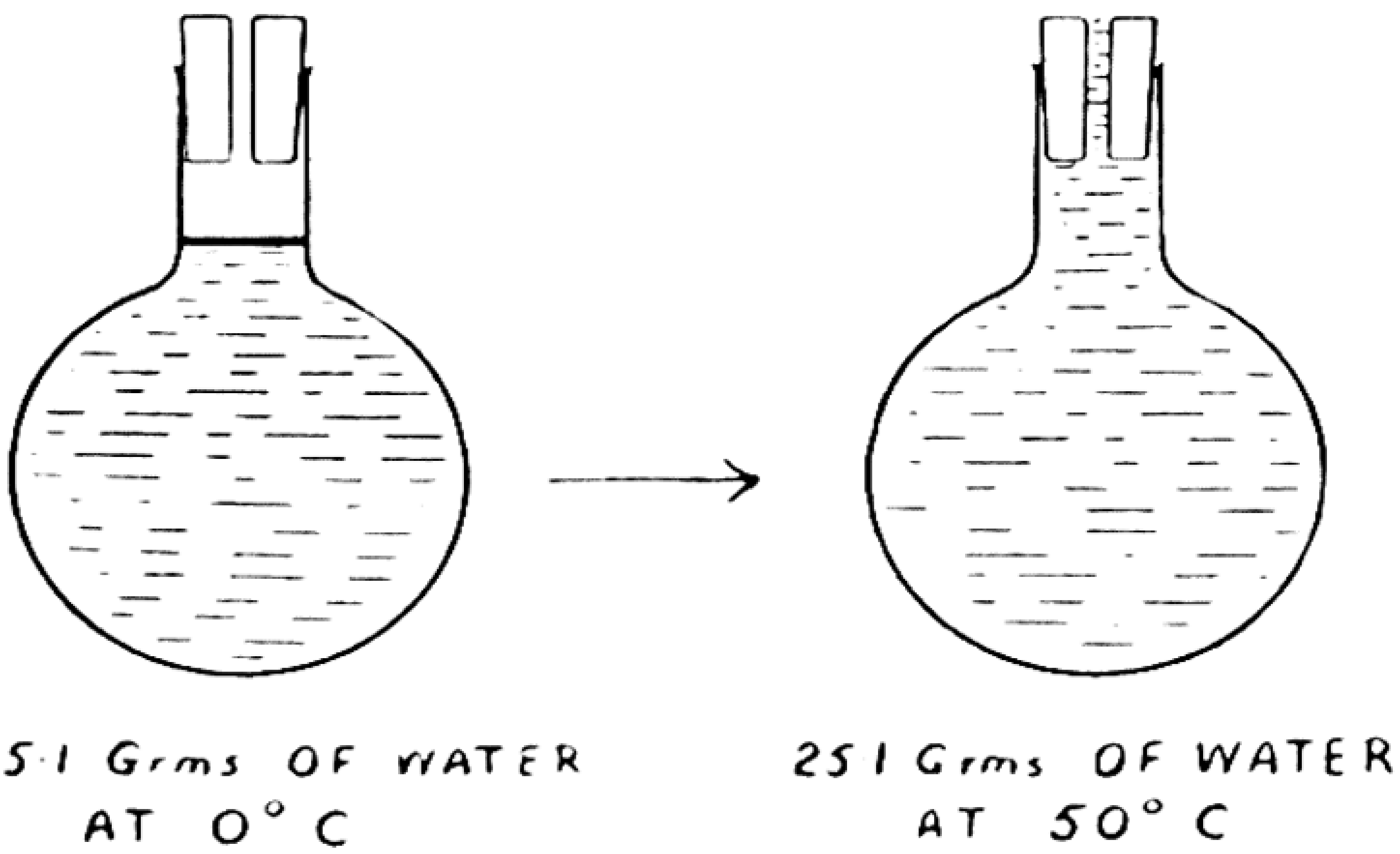


FIG. 11.—EXPANSION OF A LIQUID.

of water from 0° to 50° C. it would once again fill the bottle (Fig. 11), and the amount of its expansion would be the same as that of the water expelled, whose weight we have found to be ·28 gm. (This argument assumes that the volume of the bottle has not changed when heated, but allowance will be made later for the fact that this is not so.) Suppose 1 gm. of water at 0° C. has unit volume. Then 25·1 vols. of water at 0° C. expand by ·28 vol. while being heated to 50° C. Hence

Unit vol. of water, heated from 0° to 50° C., would expand

by $$\frac{\cdot 28}{25 \cdot 1}$$

and unit vol. of water, heated from 0° to 1° C., would

expand by $$\frac{\cdot 28}{25 \cdot 1 \times 50}$$
$$= \cdot 00022 \text{ of its volume.}$$

This result, ·00022 vol., being the amount by which unit volume of water expands when heated 1° C., is the *apparent coefficient of expansion of water*, when heated in a vessel of the material used in this experiment. The result would be different if a vessel made of other material were used. If the real coefficient of expansion of the glass bottle is taken as ·000024, then—

Real coefficient of expansion of water between 0° and 50° C. = ·00022 + ·000024 = ·000244.

To give this answer to 6 decimal places implies greater accuracy than is normally obtainable in this experiment. The last figure should therefore be neglected.

To Find the Real Coefficient of Expansion of a Liquid

This apparatus was first used by the Frenchmen Dulong and Petit, and consists primarily of a large U-tube in which is placed the liquid to be examined (Fig. 12). One of the limbs is steam-jacketed, while the other is kept at the temperature of melting ice by passing a stream of ice-cold water round it. The horizontal tube joining the two limbs is of narrow bore, so that any change of pressure in one limb is transmitted to the other, but the amount of heat conducted from the hot to the cold limb is kept to a minimum. The liquid most commonly examined by this method is mercury. It will be clear that steam-jacketing could only be used for a liquid with a boiling point well above that of water.

The pressure (that is, the force per unit area) at all points in the same horizontal plane in a liquid at rest is the same. Hence the pressure at each of the points X and Z in the horizontal tube will be the same, despite the height of liquid

in the hot tube being greater than that in the cold tube. This is because the liquid in the hot tube is less dense than that in the cold. The pressure in a liquid of density D at a depth H below its surface is HD. So if the densities of the liquid in the cold and hot limbs are D_1 and D_2, and if the heights of liquid are respectively H_1 and H_2, then pressure at $X = H_1D_1 =$ pressure at $Z = H_2D_2$. Now, equal

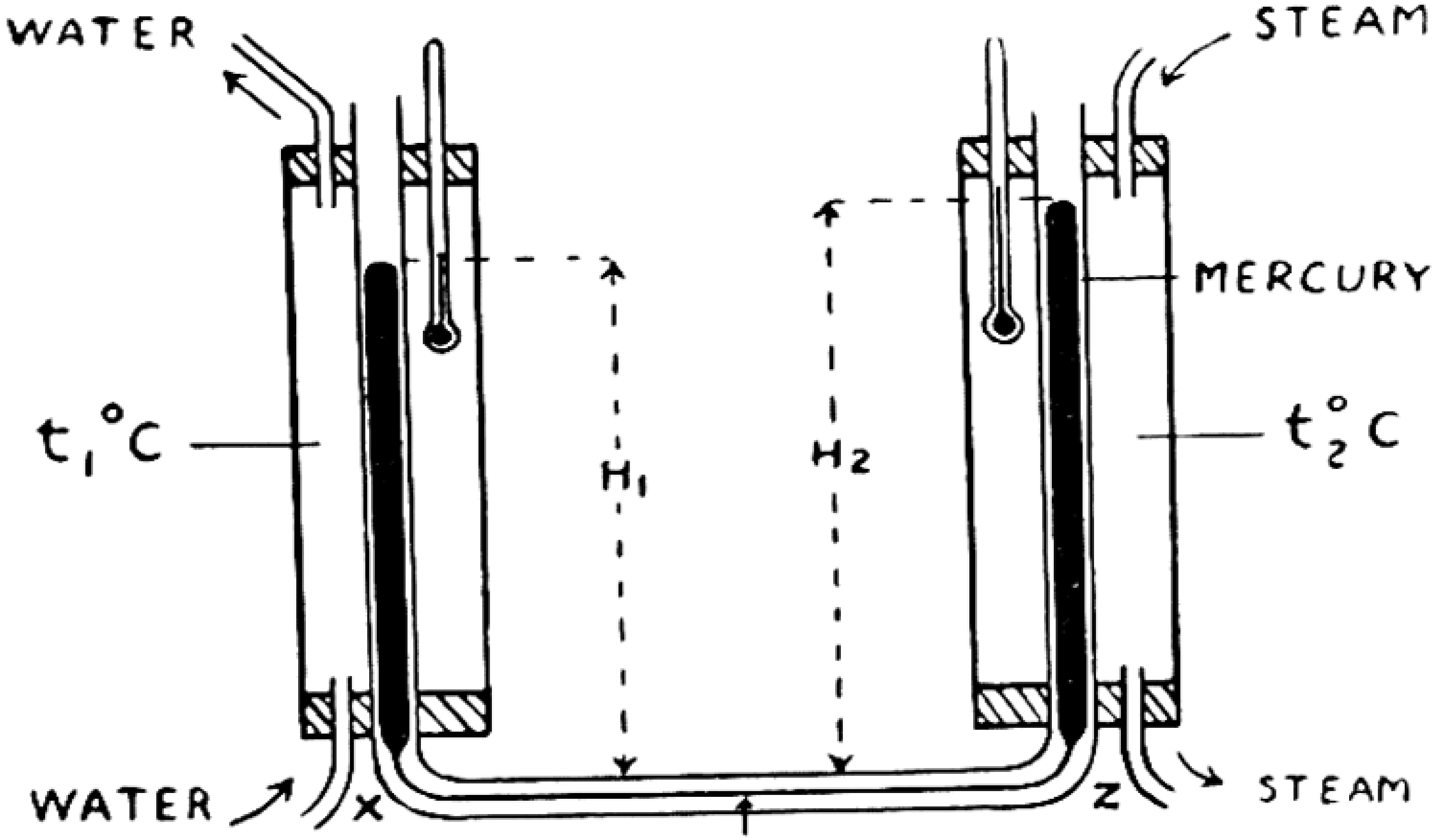


FIG. 12.—EXPANSION OF MERCURY.

masses, M, of this liquid are at the temperatures t_1 and t_2 respectively of the cold and hot limbs, and have volumes V_1 and V_2. Then

$$D_1 = \frac{M}{V_1} \text{ and } D_2 = \frac{M}{V_2}*$$

So we can say that

$$H_2D_2 = H_2 \cdot \frac{M}{V_2} = H_1D_1 = H_1 \cdot \frac{M}{V_1}$$

Dividing by M, $H_2V_1 = H_1V_2$

and
$$\frac{H_2}{H_1} = \frac{V_2}{V_1}$$

* See 'Teach Yourself Mechanics,' p. 234.

Let α = real coefficient of expansion of liquid

Then $V_2 = V_1[1 +$
α (temperature difference between V_1 and V_2)]

$= V_1[1 + \alpha(t_2 - t_1)]$

$$\therefore \quad \frac{H_2}{H_1} = \frac{V_1[1 + \alpha(t_2 - t_1)]}{V_1} = 1 + \alpha(t_2 - t_1)$$

$$\therefore \quad \alpha(t_2 - t_1) = \frac{H_2}{H_1} - 1 = \frac{H_2 - H_1}{H_1}$$

$$\therefore \quad \alpha = \frac{H_2 - H_1}{H_1(t_2 - t_1)}$$

that is, real coefficient of expansion of liquid

$$= \frac{\text{Difference of heights of liquid in limbs}}{\text{Height of liquid in cold limb} \times \text{Temperature difference between limbs.}}$$

(The form of apparatus differs in practical details so as to bring the liquid levels in the limbs in closer proximity to measure $(H_2 - H_1)$ more accurately, but the method is basically that described.)

Knowing the real coefficient of expansion of mercury by this experiment, the apparent coefficient can be determined by the specific-gravity-bottle method. Then, by subtracting the latter result from the former, the real coefficient of expansion of the material of the bottle can be found. When this has been found it can be added to the results obtained for the apparent coefficients of other liquids and the value of the real coefficients of expansion of those liquids determined.

Expansion of Gases

The expansion of gases is much greater than that of liquids, which themselves expand more than solids, the change of temperature being assumed to be the same in each case. Thus the coefficient of cubical expansion of copper is ·00005 per degree C., of mercury ·00018, and of air ·00367. There is a considerable difference between the coefficients of expansion of different solids, and the same is true for different liquids.

The coefficient of expansion of all gases, however, is the same, a remarkable result when the great diversity of gases which exist is considered. Some idea of the reason for this difference can be obtained from the fact that when substances are heated they are acted on by two forces having opposite effects. The additional energy gained when they become hotter makes the molecules of which they are composed travel more swiftly, and, in the case of gases, move farther apart. Acting against this force, however, is the force of mutual attraction between molecules, which increases very rapidly (up to a point) as the distance between them decreases. In the case of solids and liquids the latter force is very important, but in gases the molecules are so much more widely separated from each other that it plays little part. With gases it is almost entirely the first force only which acts, and this is the same for all molecules. Hence the uniformity in the expansion of gases.

Boyle's Law * states that "*if the temperature of a fixed mass of a gas is constant, then the pressure and volume of the gas are inversely proportional.*" That is, if the pressure is doubled, the volume is halved, and vice versa.

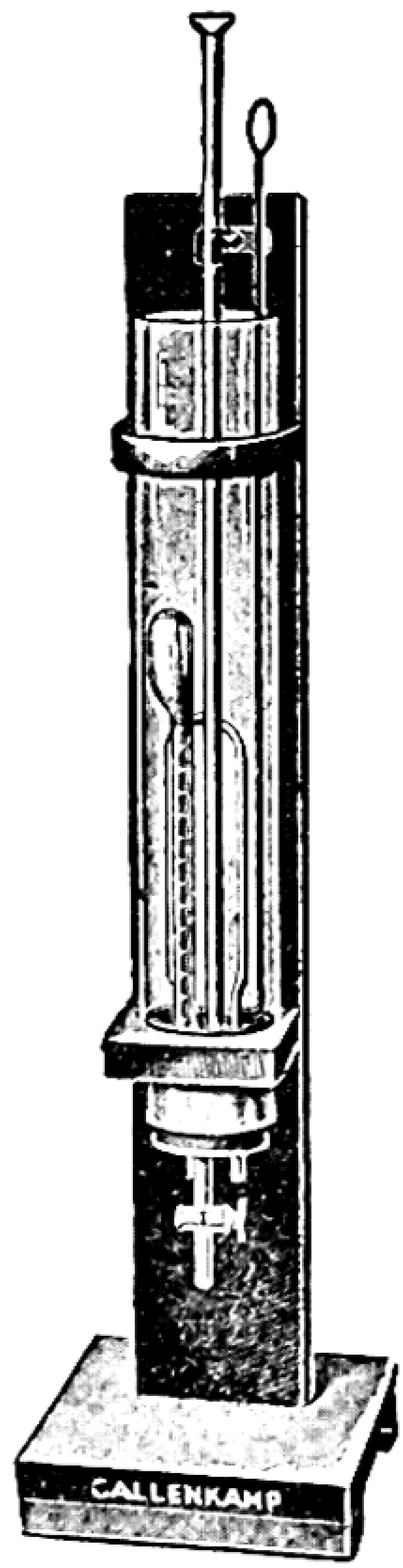


FIG. 13.—EXPANSION OF A GAS.

At present, however, we are concerned with the behaviour of a gas, which has its temperature raised, while being kept at constant pressure. This can be studied by means of the apparatus, made by GALLENKAMP, shown in Fig. 13. It consists of a U-tube with one limb closed, and graduated in cubic centimetres. It will be shown later that air saturated

* See 'Teach Yourself Mechanics', p. 254.

with water vapour obeys different laws from dry air. For this reason the liquid used to isolate a volume of air in the closed limb is concentrated sulphuric acid instead of mercury, which is frequently used in volumetric gas work.

The acid will ensure that the air is dry. The vessel surrounding the U-tube is filled with water, ice added, and the whole thoroughly stirred at frequent intervals. After a few minutes the water will be at the temperature of melting

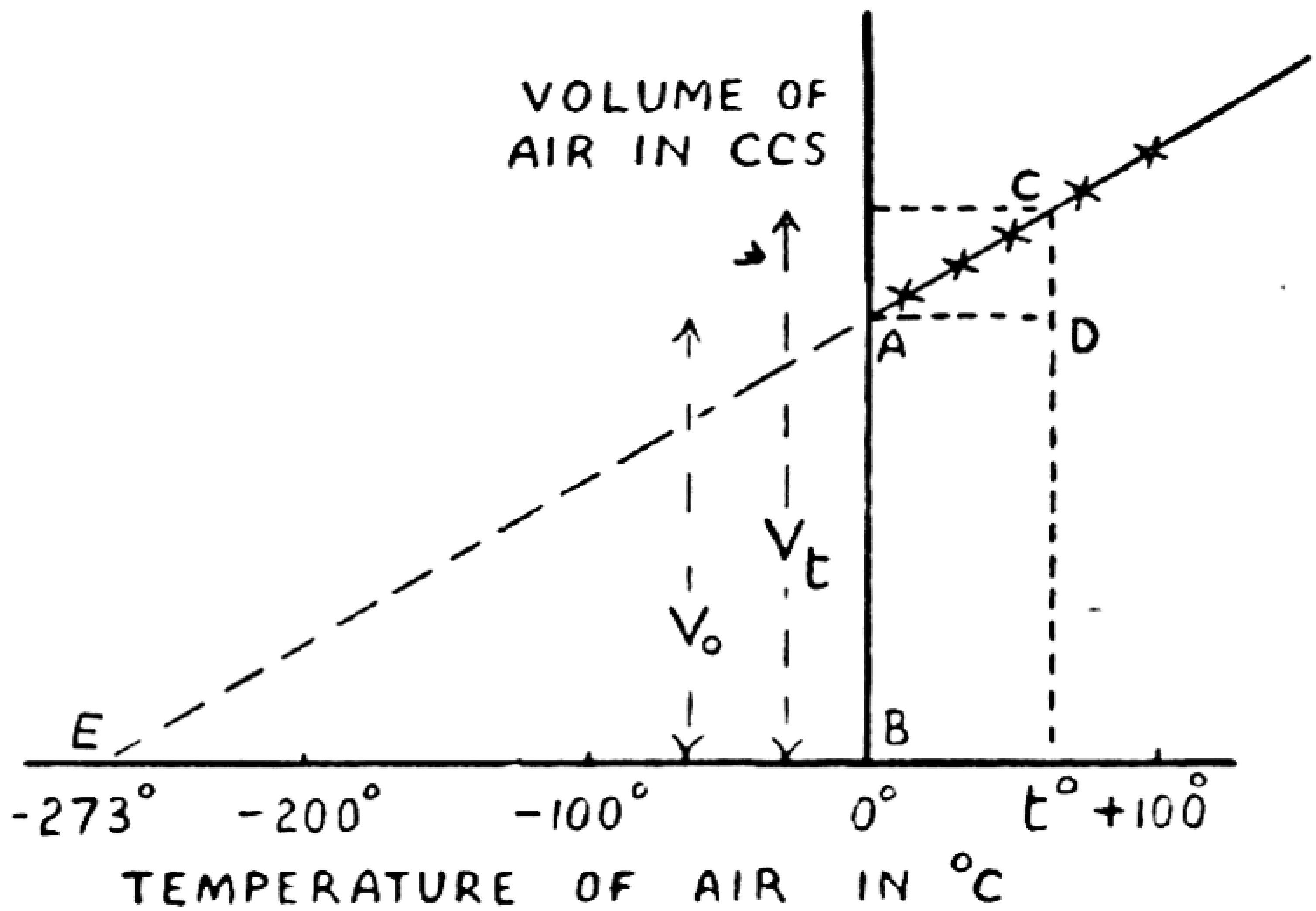


FIG. 14.—CHANGE OF VOLUME OF AIR WITH TEMPERATURE.

ice. Acid should then be drawn from the tube by means of the tap at the bottom until the levels in the two limbs are the same. Since the pressure at the surface of the acid in the open limb is that of the atmosphere, the pressure of the air enclosed in the other limb must then also be atmospheric. The volume of the enclosed air and the temperature of the air (assumed to be the same as that of the surrounding water) are then noted. The water is heated by an electric immersion heater or, less elegantly, by replacing some of the

cold water by hot water, or again by passing in steam. Regular stirring must be continued throughout the liquid. When the temperature has risen to about 10° C., again bring the acid in each limb to a common level, and note the volume and temperature of the air. Continue the observations as the temperature rises, at intervals of about 10° C., up to the boiling point of water. Now plot the volumes of the air against its temperature on squared paper. It will be found that a straight line can be drawn through the points (see Fig. 14). At 0 C°., as we found in the experiment, the gas has a certain volume, but if the graph is produced backwards to lower and lower temperatures it will be found that at the temperature of −273° C. the gas would, in theory, occupy no volume at all. Actually air, and all other gases, liquefy at higher temperatures than this.

In Fig. 14 consider any temperature $t°$ C., between 0° and 100°. Let V_0 and V_t be the volume of the air in the closed limb at 0° and $t°$ respectively. Since the graph is a straight line, its slope must be the same throughout its length, and is represented by $\frac{AB}{EB}$ or $\frac{CD}{AD}$, because the triangles EAB and ACD are exactly similar in shape, and differ only in size.

Since $\frac{CD}{AD} = \frac{AB}{EB}$ we can substitute for the lengths of these sides in terms of V_0, V_t, $t°$ and 273°, and we find that

$$\frac{V_t - V_0}{t} = \frac{V_0}{273}.$$

Multiplying each side of this equation by t

$$V_t - V_0 = \frac{1}{273} V_0 t$$

so

$$V_t = V_0 \left(1 + \frac{1}{273} t\right).$$

This equation enables us to calculate the volume of a fixed mass of any gas at any temperature, provided we know its volume at another temperature. Thus, if the volume of a

fixed mass of a gas is 100 c.c. at 0° C., what is its volume at 60° C.? Here $V_0 = 100$ c.c., $t = 60°$ C., and V_t is to be found.

$$\text{So} \qquad V_t = 100\left(1 + \frac{1}{273} \times 60\right) = 100\left(\frac{333}{273}\right)$$
$$= 122 \text{ c.c.}$$

It may be pointed out that the general equation can be written in another form which makes it easier to use and demonstrates more clearly that $-273°$ C. is the absolute zero of temperature. For we can put

$$V_t = V_0\left(\frac{273 + t}{273}\right).$$

This is known as **Charles' Law**.

Now $(273 + t)$ is a measure of the number of degrees the temperature $t°$ C. is warmer than the absolute zero of temperature. Temperatures which are measured from the absolute zero are called **absolute temperatures** and are expressed in *degrees Kelvin* (° K.) or *degrees absolute* (°A.), to distinguish them from the Centigrade system. (The name of Kelvin is applied because it was Lord Kelvin who first advocated the use of the absolute scale of temperature.) The temperature change represented by one degree is the same in both systems. The difference is in the zero of the absolute scale being 273° below that of the Centigrade scale.

We can now express Charles' Law in words: *The volume of a fixed mass of any gas at constant pressure is proportional to its absolute temperature.*

We have studied the effect of temperature on the volume of a fixed mass of gas kept at constant pressure. What happens to the pressure of a fixed volume of gas when its temperature is altered? The apparatus of Fig. 15 (known as an *air thermometer*) assists us to find the answer to this question. It consists of a bulb B containing air which is kept dry by the presence of a few drops of strong sulphuric acid, and which can be heated in a water-bath. B is connected to the tubes C and E, which are joined by the rubber

tubing D and contain mercury. E can be raised or lowered with reference to C, and a vertical scale between them allows the difference in height H between the mercury level in C and E to be measured. Inside the tube C is a vertical ivory peg with a sharp point. By bringing the mercury level always just into contact with this peg, the volume of air under examination is kept constant.

Measure the atmospheric pressure h in cm. of mercury by

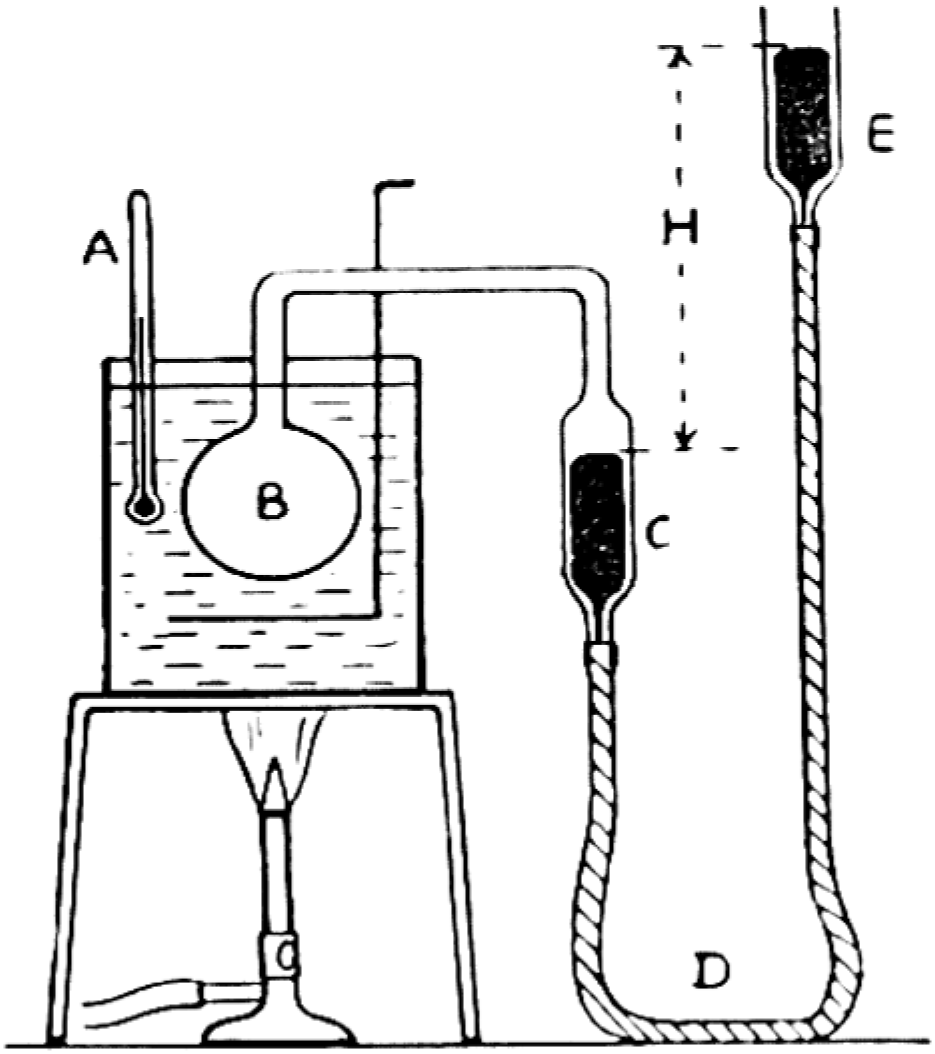


FIG. 15.—AIR THERMOMETER.

means of a barometer. Commence with the water in the bath A at room temperature. Adjust the height of the mercury in E, by raising or lowering that tube, until the surface of the mercury in C just touches the bottom of the peg. Measure the difference (H cm.) in height between the mercury levels in the two tubes, and note the temperature in the water-bath, which is taken as being that of the air in B. The pressure at the surface of the mercury in E is that of the atmosphere, which is equivalent to that of h cm. of mercury. At a point in the mercury H cm. vertically

below the surface, the pressure will clearly be that due to $(h + H)$ cm. of mercury. Now the pressures at all points in the same horizontal plane in a liquid at rest are equal. Hence the pressure at the mercury surface in C, which is also that of the air in B, is equal to that at the same horizontal level in E, and so is equal to that of $(H + h)$ cm. of mercury. If it is necessary to make the level in E, H cm.

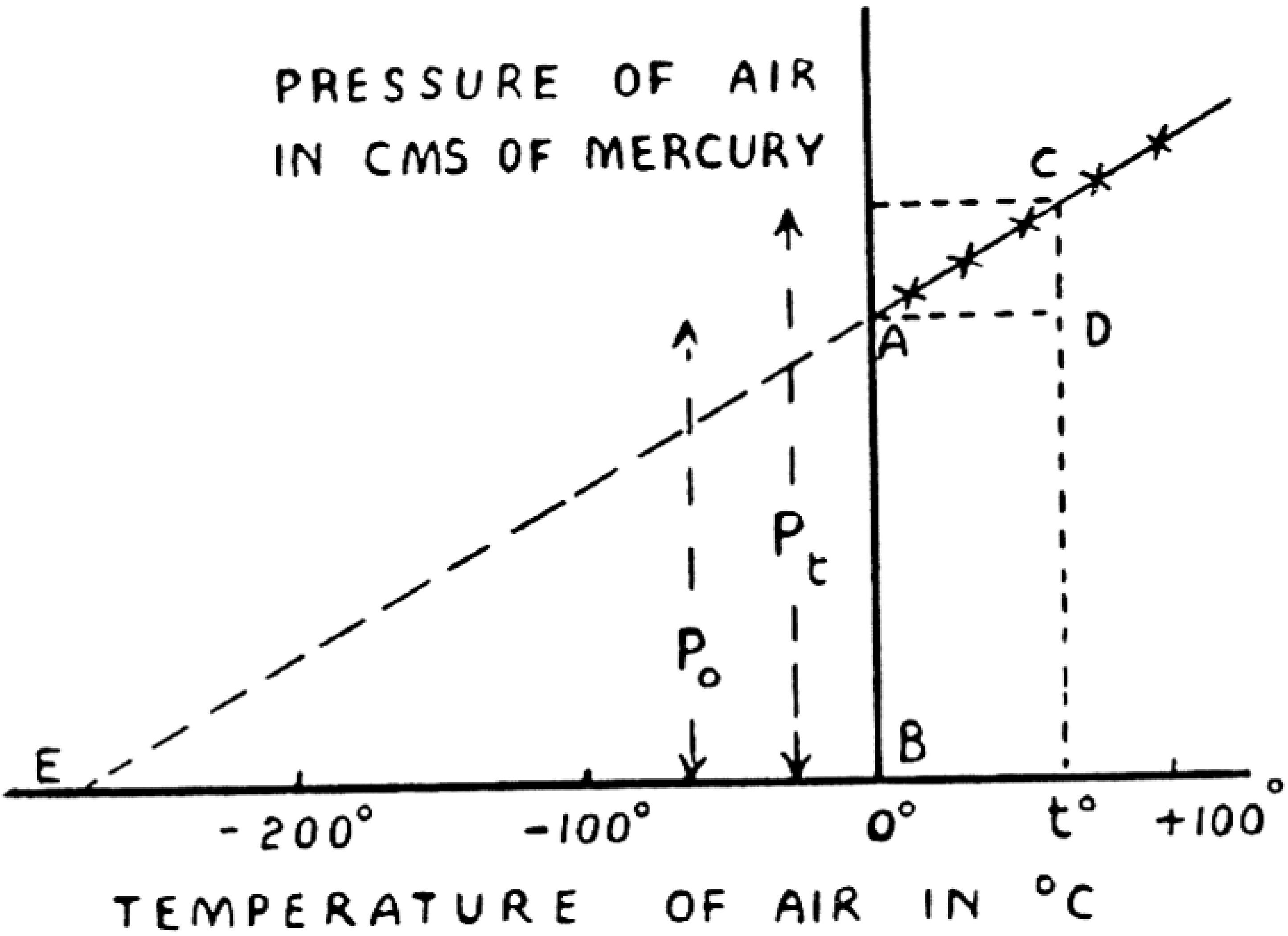


FIG. 16.—CHANGE OF PRESSURE OF AIR WITH TEMPERATURE.

lower than that in C to bring the latter level to the bottom of the peg, then by a similar argument the pressure of the air in A will be $(h - H)$ cm. of mercury.

Heat the water in the water-bath from room temperature to boiling point, and at intervals of 10° C. raise the tube E so as to bring the mercury level in C back to the standard position. Note the value of H and the temperature t° C. of the water each time. It is only justifiable to assume that the temperature of the air in B is the same as that of the water, at any given instant, if the rate of rise of temperature is slow at the time when the reading is taken. This can be

done by turning the bunsen low shortly before the desired temperature is reached.

Now tabulate the values of the pressure $(h + H)$ against those of the temperature $t°$ C. thus:

Atmospheric pressure (h) = cms. of mercury.

Temperature, $t°$ C.	Additional head (H) in cm. of mercury.	Total pressure $P_t = (h + H)$ in cm. of mercury.

Draw a graph showing the connection between the total pressure P_t $(= h + H)$ and the temperature $t°$ C. It will be found to be a straight line, showing that there is a linear relation between the pressure of air and its temperature when the volume is kept constant (Fig. 16).

We can apply to Fig. 16 similar reasoning to that used with Fig. 14. As before, $\frac{CD}{AD} = \frac{AB}{EB}$, and by substitution we get:

$$\frac{P_t - P_0}{t} = \frac{P_0}{273}.$$

Multiplying each side of this equation by t we get:

$$P_t - P_0 = \frac{1}{273} \cdot P_0 \cdot t$$

So
$$P_t = P_0\left(1 + \frac{1}{273}t\right).$$

This equation can be expressed as follows:

If a fixed mass of gas kept at constant volume is heated through 1° *C., then its pressure is increased by one two-hundred and seventy-third part of its pressure at* 0° *C.* The fraction $\frac{1}{273}$ is called the coefficient of increase of pressure.

As an example of the use of this equation, a litre bottle

of air at a pressure of 760 mm. of mercury and 15° C. is sealed and then heated to 30° C. What will be the pressure inside the bottle at the higher temperature?

Note that we are not given the pressure of the air at 0° C. It is not necessary to find it, for:

$$P_{15} = 760 = P_0\left(1 + \frac{1}{273} . 15\right) \quad . \quad . \quad . \quad (1)$$

$$P_{30} = P_0\left(1 + \frac{1}{273} . 30\right) \quad . \quad . \quad . \quad (2)$$

Dividing (2) by (1):

$$\frac{P_{30}}{760} = \frac{1 + \frac{30}{273}}{1 + \frac{15}{273}} = \frac{303}{288} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(Note that 303° and 288° on the absolute scale are the same as 30° and 15° C.) From (3):

$$P_{30} = 760 \times \frac{303}{288} = \underline{799{\cdot}6 \text{ cm.}} \text{ of mercury.}$$

This result is true in general, namely, that *the pressure of a fixed mass of gas kept at constant volume is proportional to the absolute temperature.* Hence if P_1 and P_2 are the pressures of such a mass of gas, kept at constant volume, at absolute temperatures T_1 and T_2, then

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}.$$

We have already found that for a fixed mass of gas at constant pressure the volume of the gas is proportional to the absolute temperature—that is, if V_1 and V_2 are the volumes at absolute temperatures T_1 and T_2, then:

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}.$$

What happens when the pressure, volume, and temperature are all varied? This is less complex than might appear at first sight, provided the changes are taken in stages. Con-

sider a fixed mass of gas in a cylinder, and let its pressure be P_1 and its volume V_1, at the absolute temperature T_1 (Fig. 17 (i)). Now slowly increase the pressure to P_2 without changing the temperature T_1, and let the new volume be V. Since the temperature has remained constant, Boyle's Law can be applied and we have:

$$P_1V_1 = P_2V$$

so that
$$V = \frac{P_1V_1}{P_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This is the state in Fig. 17 (ii). Keeping the pressure con-

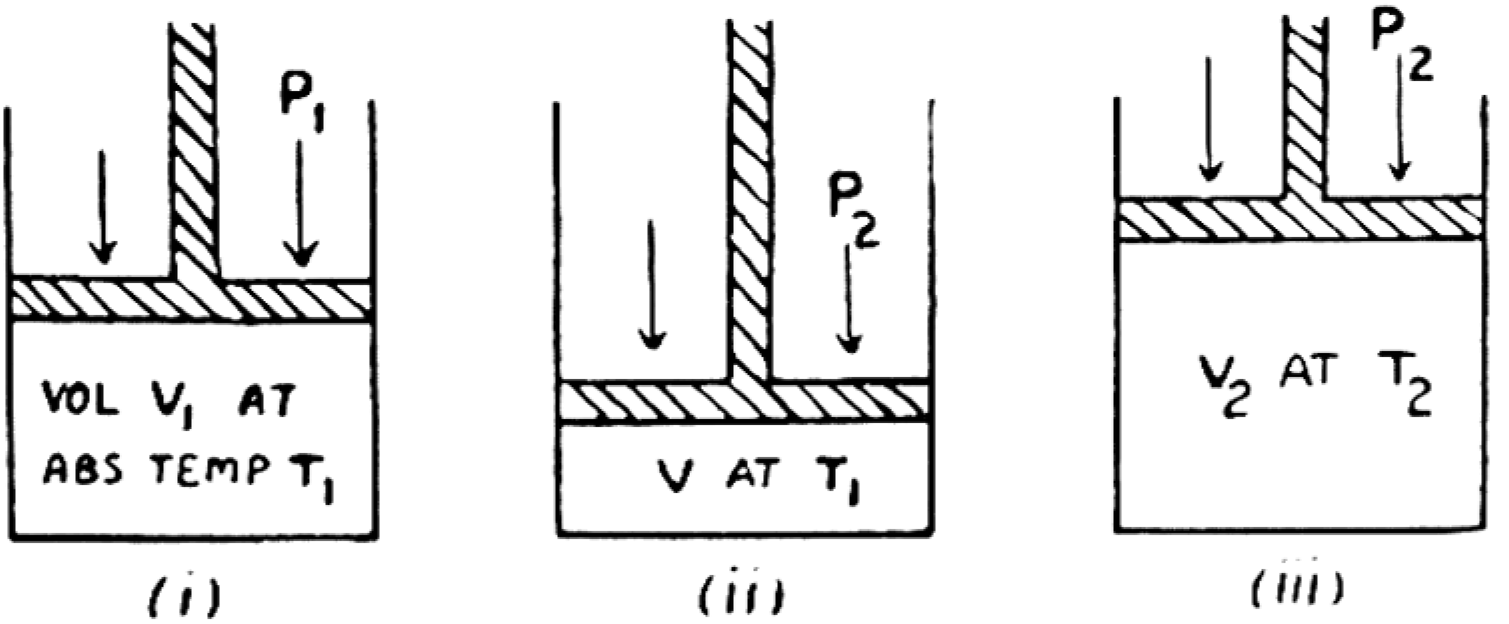


FIG. 17.—EXPANSION OF AIR.

stant, change the absolute temperature of the gas to T_2 and let the new volume be V_2 (Fig. 17 (iii)). Then from Charles' Law:

$$\frac{V}{V_2} = \frac{T_1}{T_2} \text{ and so } V = \frac{V_2T_1}{T_2} \quad . \quad . \quad . \quad (5)$$

Equating the values of V in (4) and (5):

$$\frac{P_1V_1}{P_2} = \frac{V_2T_1}{T_2}$$

$$\therefore \quad \frac{P_1V_1}{P_2V_2} = \frac{T_1}{T_2}.$$

This equation shows that if the conditions of pressure, volume, and absolute temperature of a fixed mass of gas are

changed, any one of these quantities can be calculated for the new state, provided the other two are known. Thus if 50 c.c. of a gas at 15° C. and 760 mm. pressure are heated to 40° C. and the pressure changed to 900 mm., what will be the new volume?

$$15^\circ \text{ C.} = 288^\circ \text{ A. and } 40^\circ \text{ C.} = 313^\circ \text{ A.}$$

$$\therefore \qquad \frac{760 \times 50}{900 \times V_2} = \frac{288}{313}.$$

$$V_2 = \frac{50 \times 760 \times 313}{900 \times 288} \quad . \quad . \quad . \quad . \quad (6)$$

$$= \underline{45{\cdot}89} \text{ c.c.}$$

The same problem can also be readily solved in stages from first principles. For if the pressure is increased at constant temperature, the volume must be reduced, so we must multiply 50 c.c. by $\frac{760}{900}$ and *not* by $\frac{900}{760}$. Similarly, if the temperature is increased at constant pressure, the volume must be increased, so we must multiply by $\frac{313}{288}$. The combined effect of the pressure and temperature changes will therefore give the same value for V_2 as in (6) above.

CHAPTER III

HOW HEAT IS MEASURED

Quantity of Heat

In Great Britain, coal and oil are our principal sources of power, the energy of rivers, tides, and the wind being little used. By whatever means their energy is finally applied, coal and oil must first be used to produce heat. It will therefore be clear that there is a very intimate connection between heat and power. Further, it becomes important to be able to measure heat, so that the efficiency with which it is turned into energy can be determined. (So far we have used the terms ' energy ' and ' power ' in the everyday sense in which they are interchangeable. The scientific definition of *energy*, however, is *capacity for doing work*, and of *power*, *rate of doing work*, that is the amount of work done in a specified time.)

There are several different units of heat, all sharing the fact that they are the amount of heat required to raise unit mass of water through 1° rise in temperature. From the viewpoint of the British engineer there are two important heat units. The first is the *British thermal unit* (B.Th.U.), which is the *amount of heat needed to raise* 1 *lb. of water through* 1° *F.* A *therm* (used in measuring the heating value of gas) is 100,000 B.Th.U.

The second is the *pound-degree C.* unit, being the *heat required to raise* 1 *lb. of water through* 1° *C.*

The third unit, used throughout the scientific world, is the *calorie.* It is the *amount of heat necessary to raise the temperature of* 1 *gram of water* 1° *C.*

Take several identical vessels, put the same weight of water into each (sufficient to half fill it) and heat the water to boiling point. Now put a thermometer in each vessel, and take the temperature at intervals of a minute for five minutes while the water cools. At each minute the temperatures of the vessels should be equal, showing that they

all cool at the same rate. Again heat the water in each vessel to boiling point. Take equal weights of different solid materials—copper, iron, lead, glass, etc. The most suitable weight to take would be about two or three times that of the water in each vessel. It is not necessary that the materials should be in a solid block. Turnings or chippings will be equally effective. Remove the source of heat from the water, and add one kind of material to each vessel. Take the temperatures of the water at minute intervals, as before, for five minutes. This time it will be found that the vessels have cooled at different rates. That containing glass will have cooled more quickly than that with the lead in it. The rates of cooling of the vessels containing copper and iron will be intermediate between those with glass and lead in them. Why has this difference appeared? It is because the substances dropped into the boiling water had to be heated from room temperature, and though they were all of the same mass and were put into equal masses of water at boiling point, *they had differing capacities for absorbing heat.*

The specific heat of a substance is *the ratio of the heat required to raise unit mass of the substance through* 1° *to that required to raise the same mass of water through the same temperature interval.* Since it is a ratio or fraction, the units occurring in the numerator will be cancelled by the same units occurring in the denominator, so that specific heat is simply a number. *It is the number of times the heat capacity of the substance is as great as that of water.* Further, since it is a number, its value is the same for a given substance whatever are the units of heat and temperature used.

The water equivalent of a body is the *product of its mass and its specific heat.* As its name implies, it is that mass of water which has the same heat capacity as the body.

Measurement of Specific Heat

How is the specific heat of a solid determined? The principle of the method of mixtures, which is much used, is as follows: A metal vessel, called a calorimeter (since it is to be used in measuring heat), is placed inside a larger

metal vessel, the interspace being packed with dry cotton-wool or felt (Fig. 18). The inner vessel is weighed, together with the metal stirrer in it. In an actual experiment the

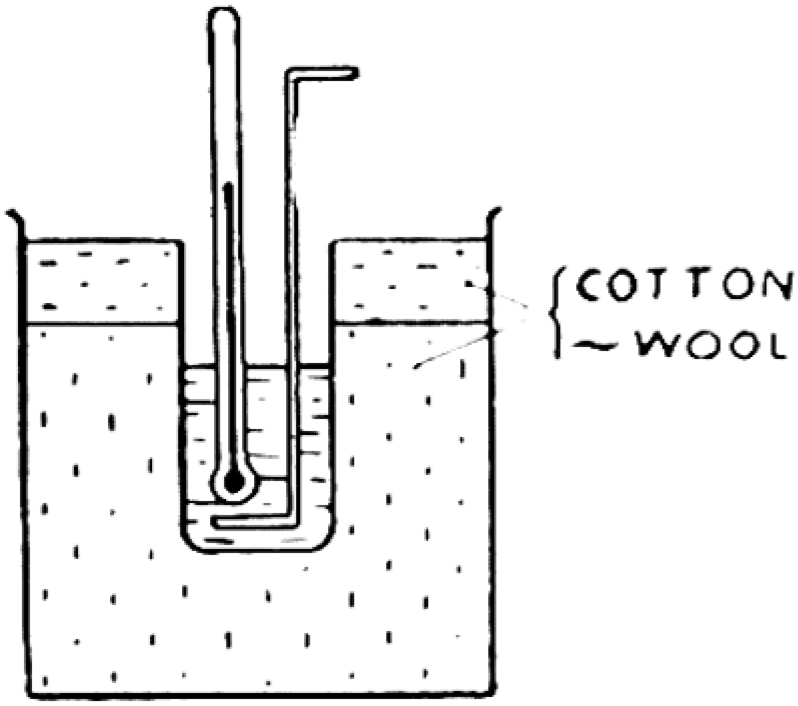


FIG. 18.—SIMPLE CALORIMETER.

vessel and stirrer, both of copper of specific heat ·095, weighed 55·2 gm. Into it was poured enough cold water to make it about two-thirds full, and on re-weighing the

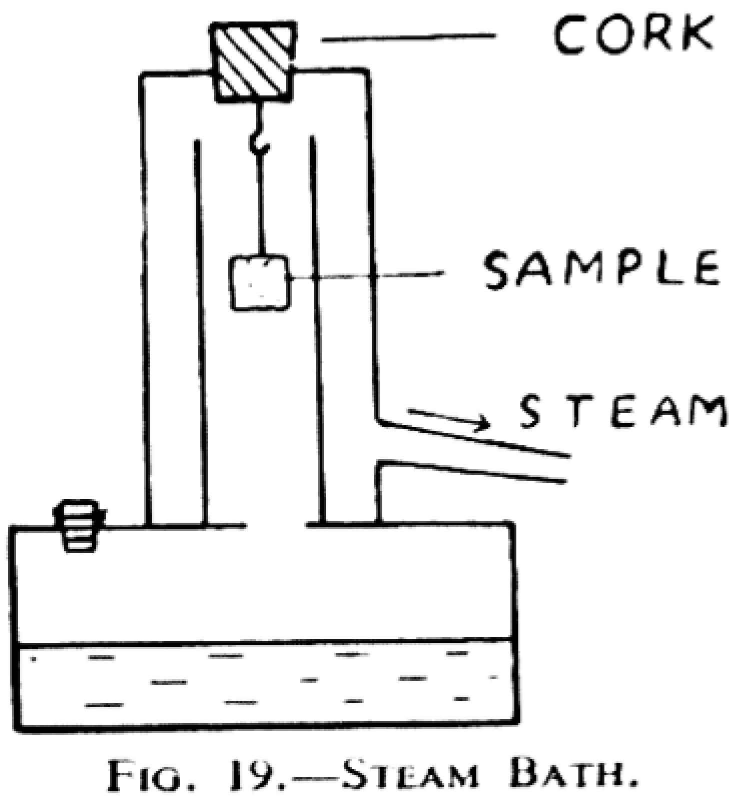


FIG. 19.—STEAM BATH.

vessel, stirrer and water weighed 148·7 gm. So the weight of water added was 93·5 gm. The solid to be tested was weighed and then lowered into a steam heater (Fig. 19) on

a piece of thread. It was allowed to remain there till it had reached the boiling point of water in temperature. The temperature of the water in the calorimeter was measured and found to be 17° C. The solid, which was a piece of iron weighing 59·7 gm., was now quickly transferred to the calorimeter, the water stirred, and the highest temperature reached observed. This temperature was 22° C. As soon as the temperature of the water was raised, it began to lose heat to the surroundings. Further, some heat would also be lost by the solid when being transferred to the water. Otherwise the temperature of the resultant mixture would have been slightly higher than 22° C. From these results the approximate value of the specific heat of the iron solid can be calculated.

We have, by definition, that 10 gm. of water heated through 1° C. would require 10 calories of heat. If heated through 5° C., $10 \times 5 = 50$ calories of heat would be needed. Now, again by definition, an equal mass of a substance of specific heat 0·2 would require $0{\cdot}2 \times 10 \times 5$ calories to be heated through the same rise in temperature. In general, therefore, a substance of mass M gm. and specific heat s heated through t° C. absorbs Mst calories. In applying this result to this experiment we shall neglect the small amount of heat lost to the surroundings and assume that :

Heat gained by	=	Heat lost by
(*a*) Water		(*c*) Iron block
(*b*) Calorimeter		

Then if s = specific heat of iron block,

$$\underbrace{93{\cdot}5 \times (22-17)}_{\text{Water}} + \underbrace{55{\cdot}2 \times {\cdot}095 \times (22-17)}_{\text{Calorimeter}} = \underbrace{59{\cdot}7 \times s \times (100-22)}_{\text{Iron}}$$

$$467{\cdot}5 + 26{\cdot}2 = 4656{\cdot}6 \times s$$

$$s = \frac{493{\cdot}7}{4656{\cdot}6} = \underline{0{\cdot}11}$$

The result is given to only two places of decimals. If reasonable care has been taken, errors of weighing should

be very small. Thus, if there was an error of ·2 gm. in the weight of the iron solid, this would still amount to only 0·33%. However, if the error in measuring the rise in temperature was 0·5°, this would amount to 10%. The measurement of temperature is therefore the weakest link in this experimental chain, and because of it, more than two significant figures should not be given in the final value.

Fig. 20 shows an improved apparatus devised by the French physicist Regnault for carrying out this experiment.

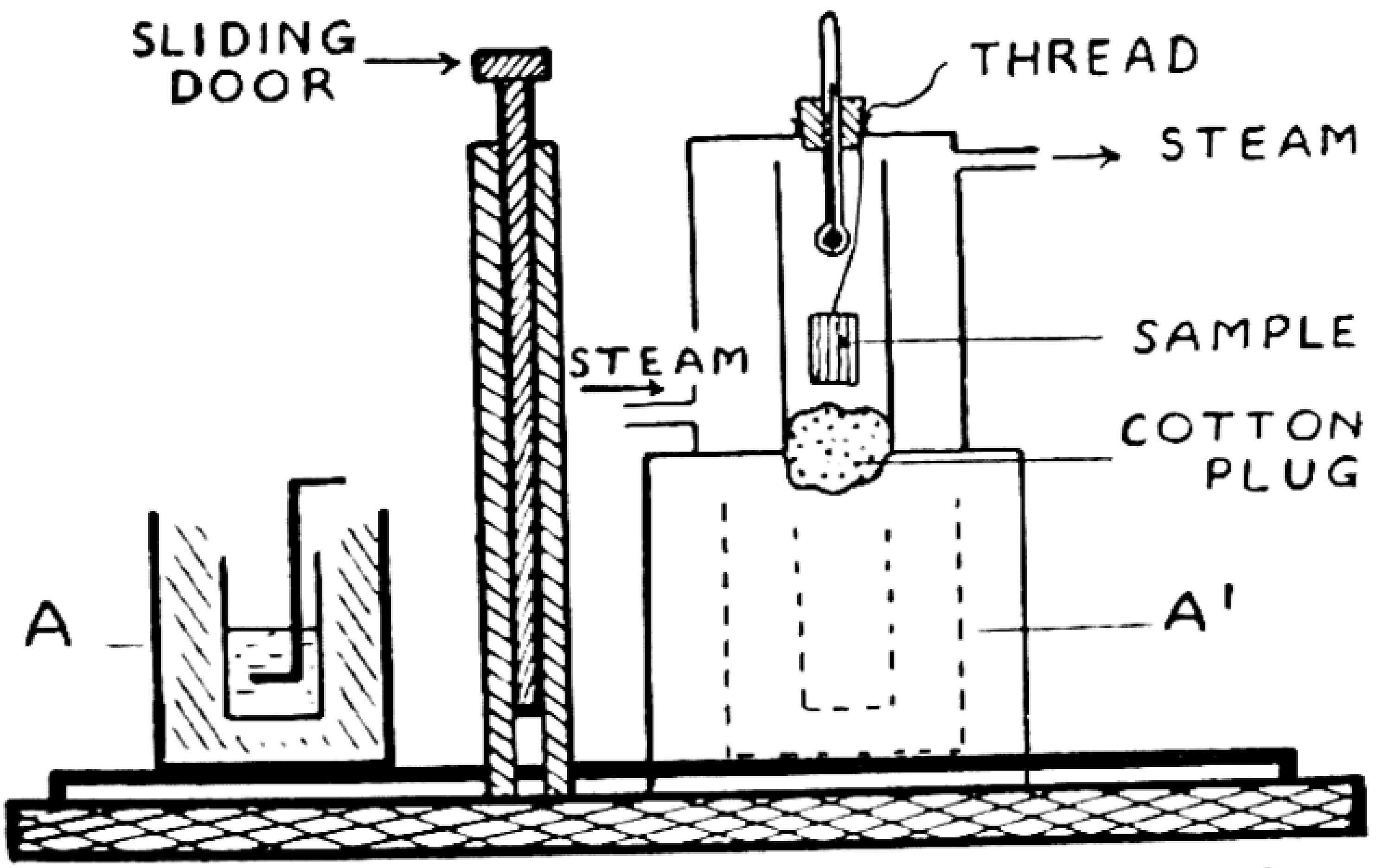


FIG. 20.—APPARATUS FOR MEASURING SPECIFIC HEAT OF SOLID.

The sample of material to be tested is heated in the steam-jacket at the right. Meanwhile the calorimeter *A*, with its known weight of water, is protected from heat emitted by the steam-jacket by means of a door or partition. The sample is kept in the steam-heater for several minutes. The temperature of the water in the calorimeter is read, the partition raised and the calorimeter slid on rails to a position *A'* below the steam-heater. The cotton plug is removed, the thread holding the sample is released so that the latter falls into the calorimeter and the calorimeter quickly returned to position *A*. The partition is lowered, the water

in the calorimeter steadily stirred and the highest temperature reached is observed. The method of calculation is unchanged.

Having found the specific heat of this iron block, it can be used to measure a high temperature, which would be out of the range of a mercury-in-glass thermometer. The block can be placed in the furnace whose temperature is required. It can then be dropped into a known mass of water in a calorimeter of known water equivalent. The amount of heat lost by the iron can be found, for it will be equal to that gained by the water and calorimeter, except for that lost to the surroundings during the transfer of the block. Now the heat lost by the block will also equal the product of its mass, its specific heat and its fall of temperature. The latter quantity is the only unknown, so it can be calculated. Having done so, if we add to it the known highest temperature of the water in the calorimeter, we have an approximate value of the temperature of the furnace. Because of heat loss to the surroundings, which is difficult to estimate, and so is neglected in elementary work, the result will be low.

In this experiment a furnace was not available, so the method was used to estimate the temperature of the hottest part of a bunsen flame. The weight of the piece of iron heated in the flame was 15·7 gm. The specific heat of iron at ordinary temperatures is 0·11, but its value varies slightly with temperature (as it does for most substances) and its mean value between 0° and 1000° C. is 0·15. The calorimeter used was of copper, and weighed 47·3 gm. Into it were placed 62·9 gm. of water at 12° C. After the red-hot iron had been dropped into the calorimeter and the water stirred, the highest temperature reached was 47° C. The water equivalent of the calorimeter was $47{\cdot}3 \times {\cdot}095 = 4{\cdot}5$ gm. approx., and hence the thermal capacity of the calorimeter and water was that of $62{\cdot}9 + 4{\cdot}5 = 67{\cdot}4$ gm. of water. We have:

Heat gained by calorimeter and water = Heat lost by iron.

Let $t°$ C. = temperature of iron just before immersion in water.

$$67{\cdot}4\,(47 - 12) = 15{\cdot}7 \times {\cdot}15 \times (t - 47)$$
$$2359 = 2{\cdot}355t - 110{\cdot}7$$
$$t = \frac{2469{\cdot}7}{2{\cdot}355}$$
$$= \underline{1050^\circ\ \text{C.}}$$

In addition to the heat loss from the hot solid during transfer, a further cause of under-estimation of the flame temperature is that the red-hot iron turns a little of the water into steam, and this, together with its heat, is lost. However, the method demonstrates how an estimate of a high temperature can be obtained by the use of simple apparatus.

Another use for a solid of known specific heat is in finding that of a liquid. The method resembles that previously used to find the specific heat of a solid, except that this time the unknown is the specific heat of the liquid, which replaces water in the calorimeter. In an experiment with methylated spirit, the calorimeter and metal block were those used in finding the specific heat of iron. The mass of methylated spirit taken was 81·4 gm., at a temperature of 16° C. The iron block was heated to 100° C. in steam before being dropped into the methylated spirit. The liquid was thoroughly stirred. The highest temperature reached was 25° C.

Heat gained by	=	Heat lost by
(*a*) Methylated spirit		(*c*) Iron block.
(*b*) Calorimeter and stirrer		

If s = specific heat of methylated spirit

$$81{\cdot}4s(25 - 16) + 55{\cdot}2 \times {\cdot}095(25 - 16) = 59{\cdot}7 \times {\cdot}11 \times (100 - 25).$$

$$s = \frac{445{\cdot}3}{732{\cdot}6} = \underline{{\cdot}61}.$$

The Specific Heat of a Liquid by the Method of Cooling

This method is interesting because it uses a new principle, If a definite volume of a liquid at, say, 30° C. is placed in a

calorimeter, with the surroundings at, say, 15° C., then the time taken for this liquid to cool to, say, 20° C. will always be the same, provided that the same calorimeter is used in the same surrounding conditions of temperature and position and that the same volume of liquid is cooling. The heat loss per minute will still be the same if another liquid is substituted for the first, provided that the volume is unchanged. In short, *the factors determining the rate of loss of heat of a liquid in a vessel* are: the nature of the surface of the vessel, its surface area, its temperature and that of its surroundings. It is not dependent on the nature of the liquid. If the same calorimeter is used in turn for two liquids of equal volume, the specific heat of one being known, then the heat lost per minute will be the same for both liquids when they are at the same temperature and it is possible to calculate the specific heat of the second liquid. Thus, to a copper calorimeter weighing 30·6 gm., 25 c.c. of water (weighing 25 gm.) were added by means of a pipette. The water was then heated to about 70° C., and the calorimeter was set on a large cork in a large vessel, so as to shield it from draughts. The time taken for the water to cool from 60° to 50° C. was measured, and found to be 160 seconds. The water was now replaced by 25 c.c. of benzene weighing 22·5 gm. and the procedure repeated. This time it took 73 seconds for the liquid to cool from 60° to 50° C.

The heat lost by the water and the calorimeter in cooling from 60° to 50° C. $= (25 + 30{\cdot}6 \times {\cdot}095)\,(60 - 50)$

$$= 279 \text{ calories}$$

$$\therefore \quad \text{Average loss of heat per sec.} = \frac{279}{160} = 1{\cdot}75 \text{ calories.}$$

This will also be the average rate of loss of heat by the benzene and calorimeter. So if s = specific heat of benzene, the heat lost by benzene and calorimeter in cooling from 60° to 50° C.

$$= (22{\cdot}5s + 30{\cdot}6 \times {\cdot}095)\,(60 - 50)$$
$$= (225s + 29{\cdot}1) \text{ calories}$$

$\therefore$ Average loss of heat per sec. $= \dfrac{225s + 29{\cdot}1}{73}$, which can be equated to 1·75 calories, above.

$$\therefore \quad 225s = 127{\cdot}75 - 29{\cdot}1 = 98{\cdot}65.$$
$$s = \underline{0{\cdot}44}$$

It is a common misconception that this method rests on Newton's Law of Cooling, which states that "the rate of fall of temperature of a body is proportional to the temperature difference between the body and its surroundings at any instant". Actually the method is independent of this law and would be correct whatever the law of cooling. Newton's law is correct only for small temperature differences (see p. 87).

The Connection between Heat and Work

In the previous chapter we emphasized the importance of the connection between heat and work. Near the end of the eighteenth century Count Rumford noticed the considerable amount of heat produced during the boring of cannon and in a test found that 26·5 lb. of cold water could be raised to boiling point in two and a half hours by this method, much to the astonishment of onlookers. It was Joule, the Manchester physicist, who was first able to answer accurately the question 'How much work must be done to produce a calorie of heat?' Work is done when the point of application of a force is moved against resistance, and the amount of the work done is measured by the product of the force and the distance moved by its point of application. Thus the work done when a force of 1 lb. weight moves a distance of 1 foot against resistance is called a *foot-pound.* This is the British unit. The fundamental metric unit is the *erg.* This is the work done when a force of 1 dyne moves a distance of 1 cm. against resistance.*

Joule's painstaking and accurate researches showed that *one calorie of heat can be produced by the expenditure of 42 million (or $4{\cdot}2 \times 10^7$) ergs of work.* On the British

* See 'Teach Yourself Mechanics', p. 181.

system, *one British Thermal Unit requires for its production 778 ft.-lb. of work.* These values are called Joule's mechanical equivalent of heat, and are denoted by the symbol J.

Joule produced heat by doing work in several different ways. Thus he caused falling weights to turn paddles

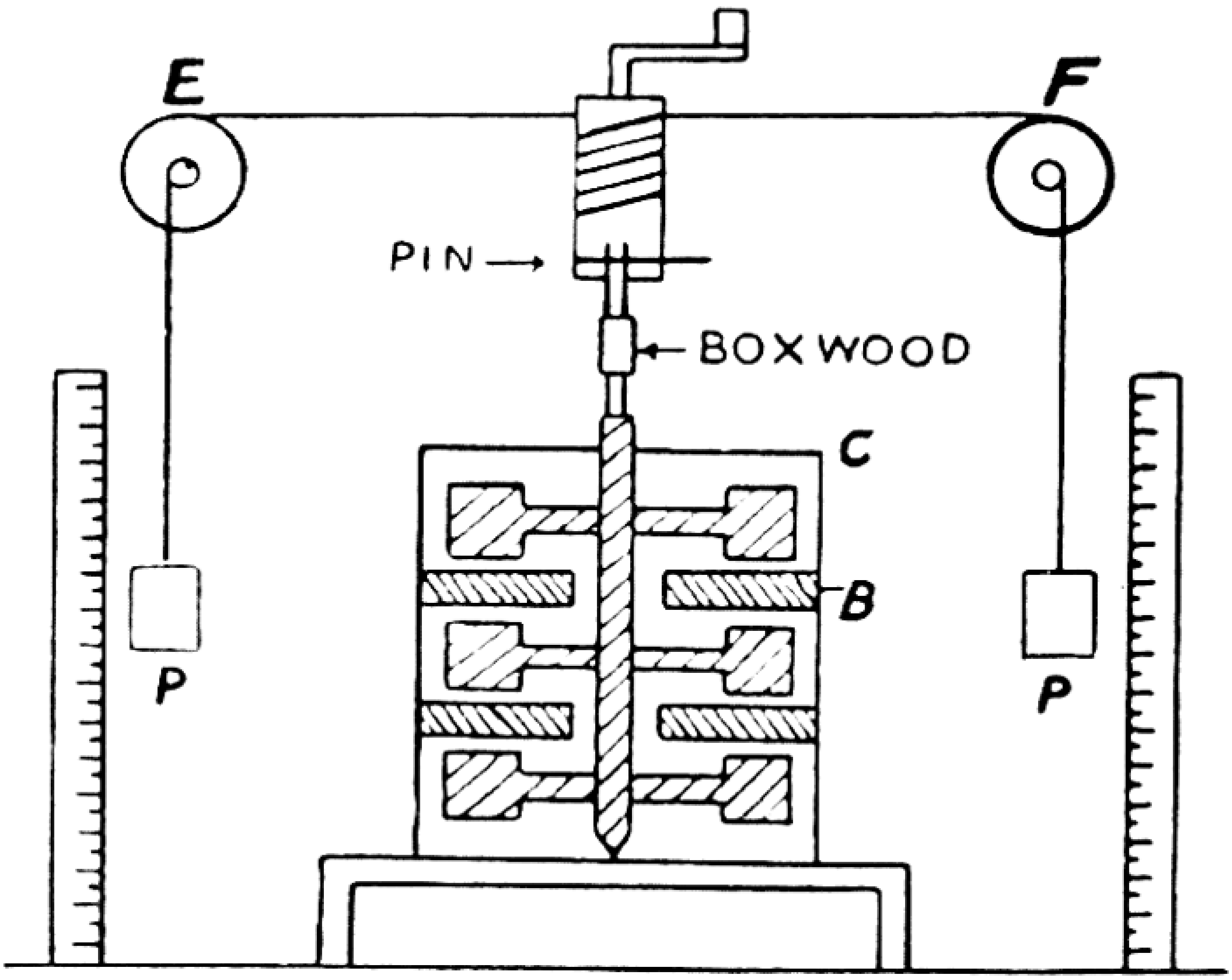


FIG. 21.—MECHANICAL EQUIVALENT OF HEAT.

churning water in a vessel, he heated water by means of a current passing through a wire, and by pressing it through tubes of small bore. In every case he measured the work W (in work units of ft.-lb. or ergs) required to produce H units of heat (in B.Th.U. or calories) and found the value of the factor J in the equation:

$$W = JH$$

Because $J = \frac{W}{H}$, it is clear that J must be in work units per heat unit—e.g., ft.-lb. per B.Th.U. or ergs per calorie.

Let us consider the first method of finding J in some detail. The apparatus is shown in Fig. 21. The calorimeter C contained baffle-plates B, so that the water was agitated by the turning paddles and not merely rotated. The weights P were allowed to fall a definite number of times and at the conclusion of each descent by removing the pin they could be wound up again by means of the handle.

If the weights were each of P lb. and fell a vertical distance h ft., then at each descent the work done by the paddles on the water was $2Ph$ ft.-lb., and if there were n descents, then the total work done was $2nPh$ ft.-lb. If the mass of water in the calorimeter was M lb. and the water equivalent of calorimeter and paddles was m lb., then if the temperature rise was $t°$ F., the heat produced was $(M + m)t$ British thermal units. Hence the equation $W = JH$ becomes:

$$2nPh = J(M + m)t,$$

and so, since J is the only unknown, it can be calculated. Joule took exceptional precautions to secure an accurate result. He made allowance for the loss of heat from the calorimeter, for the fact that the weights hit the ground at the end of each descent and so deprived the paddles of some of the $2nPh$ ft.-lb., for the work wasted in the friction of the pulleys E, F, and even for the work expended in the noise made during the rotation of the paddles! This latter was estimated by measuring the work required to be done to make a violoncello produce a sound of intensity equal to that of the whirring paddles!

The following experiment, though not giving a highly accurate result, illustrates well the principle of methods of determining J and uses simple and easily available apparatus. The requirements are a long, narrow glass or cardboard tube (about 100 cm. long and 5 cm. diameter) and two corks to fit its ends, a quantity of lead shot and a sensitive thermometer. A quantity of lead shot is introduced gently into the tube at one end. After a short delay to allow the dissipation of any heat which may have been produced by friction between the shot, their temperature is taken with the thermometer. A cork is inserted in each

end, and the tube slowly turned into the vertical position. If the tube is now inverted, the average distance fallen by the shot will not be the internal distance between the corks, but that distance less the depth to which the shot lie in the tube. Measure this distance. Now, gripping the tube firmly about the middle, invert it smartly about 50 times, counting the number of inversions. Having laid the tube horizontally, as quickly as possible remove one of the corks and measure the new temperature of the shot. The results of such an experiment were as follows. Let the mass of lead shot be M gm. Later it will be seen that M cancels out. The average height fallen by the shot at each of 50 inversions was 82·6 cm. Hence the work done on the lead shot was $50 \times M \times 82{\cdot}6$ gm.-cm. units. Now a force of 1 gm.-wt. is equal to that of 981 dynes.* So 1 gm.-cm. unit of work equals 981 dyne-cms. or 981 ergs.

Hence the work done was $50 \times M \times 82{\cdot}6 \times 981$ ergs. The temperature rise was 2·9° C. The specific heat of lead is ·032, so the heat developed was $M \times {\cdot}032 \times 2{\cdot}9$ calories. Hence in the equation:

$$W = JH$$
$$50 \times M \times 82{\cdot}6 \times 981 = J \times M \times {\cdot}032 \times 2{\cdot}9$$
$$J = \frac{50 \times 82{\cdot}6 \times 981}{{\cdot}032 \times 2{\cdot}9}$$
$$= 4{\cdot}35 \times 10^7 \text{ ergs per calorie.}$$

Change of State

If some ice is placed in water at ordinary temperatures, the ice will begin to melt. If a thermometer is placed in the water, it will indicate the melting point of ice. Now gently warm the ice and water, stirring to keep the temperature uniform throughout, and observe the temperature frequently. It will be found that so long as any ice remains, the temperature will not rise above the melting point. Heat has been supplied, yet the temperature has not risen during this period. How has it been used? It has been used in changing the state of the ice into water at the same temperature. *The heat required to change the state of a body from*

* See 'Teach Yourself Mechanics,' p. 166.

a solid to a liquid, or from a liquid to a vapour, is called **latent heat**, because it appears to lie hidden in the material.

The heat required to melt unit mass of ice into water at the same temperature is called the **latent heat of fusion** of ice. The heat required to convert unit mass of water into steam at the same temperature is called the **latent heat of vaporization** of water, or simply the latent heat of steam.

The following experiment enables the latent heat of fusion of ice to be determined. Weigh a calorimeter and stirrer, of material of known specific heat. Half fill the calorimeter with water warmed to about 30° C. and re-weigh. Note the temperature of the water. Now add, a few at a time, some small pieces of ice which have been dried on blotting-paper, so that the total mass added is entirely ice. Stir frequently till all the ice is melted. Then note the temperature. Weigh the calorimeter, stirrer, water and ice. By subtracting the previous weighing, the weight of ice added can be found.

The method of calculating L, the latent heat of fusion of 1 gm. of ice, will be best shown by an example. Thus a copper calorimeter and stirrer weighed 43·9 gm. When half filled with water it weighed 84·2 gm. Hence the weight of water added was 40·3 gm. The temperature was 28° C. The ice was added, and when it had all melted the temperature was 12° C. The final weight of the calorimeter and contents was 92·0 gm. Hence the weight of ice added was 7·8 gm.

We have:

Heat gained	=	Heat lost
by (*a*) ice in melting		by (*c*) calorimeter
(*b*) water formed		(*d*) water.

$$7{\cdot}8 \times L + 7{\cdot}8\,(12 - 0) = (43{\cdot}9 \times {\cdot}095 + 40{\cdot}3)\,(28 - 12)$$
$$7{\cdot}8L + 93{\cdot}6 = 711{\cdot}5$$
$$L = \underline{79{\cdot}2\ \text{cal.}}/\text{gm.}$$

(The accepted value of L is 80 calories per gram.)

The Melting Point of a Solid

The latent heat which is absorbed when the solid melts is given out again when it solidifies once more. Thus when a

cooling liquid reaches the melting point, its rate of cooling is temporarily arrested until all the latent heat has been emitted. This fact is often used to find the melting point of a solid. Naphthalene is a convenient substance with which to demonstrate this method. Place some of it in a wide test-tube fitted with a cork having a thermometer passing through it and dipping into the solid. An additional hole in the cork will facilitate cooling when required.

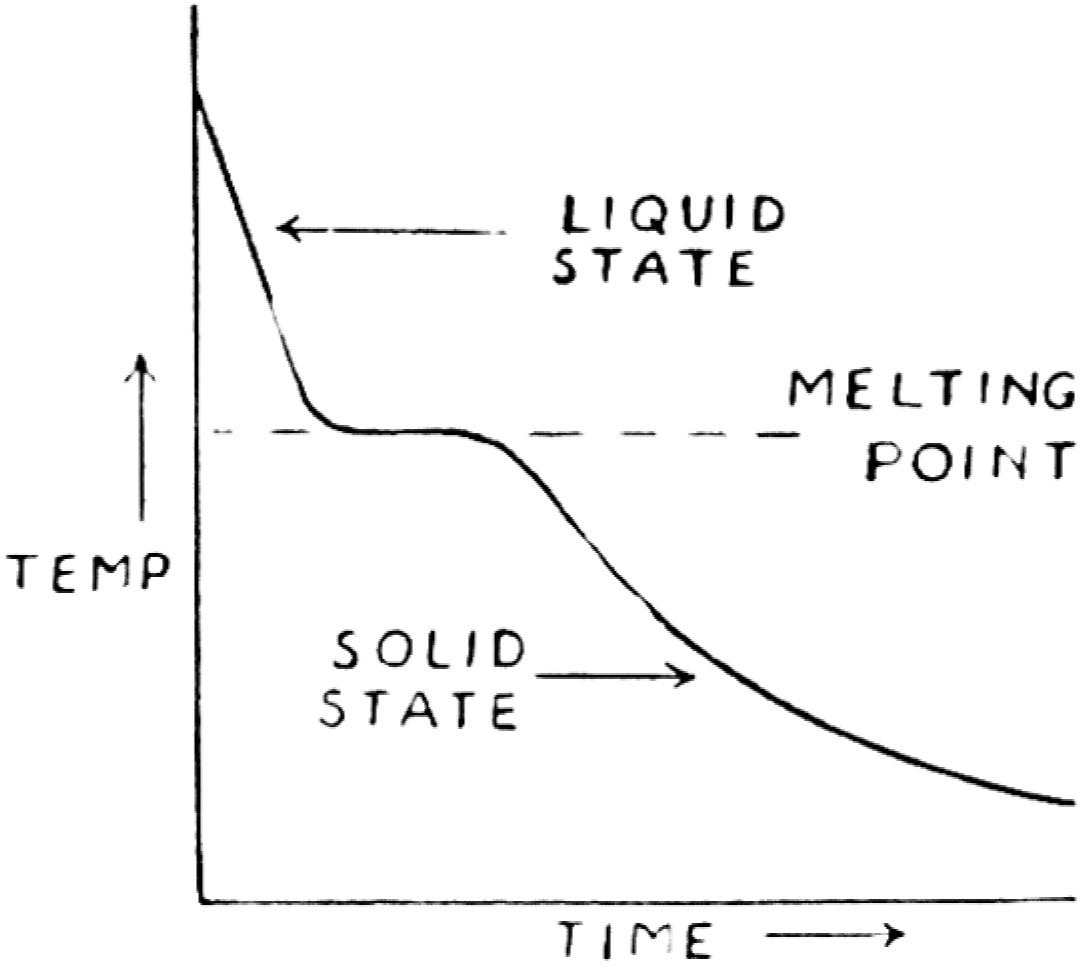


FIG. 22.—COOLING CURVE.

Fix the tube in a clamp. Warm the tube with a gas flame, continuing till a short time after all the naphthalene has been melted. Remove the flame, and begin to observe the temperature of the liquid every half-minute, until it approaches that of the room. Plot the temperature on a graph against time. The result will be as in Fig. 22. The melting point is that at which the temperature, for a short period, remains constant. If a mixture of substances is used, the solidifying point is less definite. For this reason, the method is sometimes used to test the purity of a substance.

The Latent Heat of Vaporization of Water

It is well known that when water is being heated and turned to steam, its temperature remains at boiling point till all the water has been evaporated. The heat passing into the water is not being used to increase its temperature, but only to change it into steam at the same temperature. The value of the latent heat of steam can be found by use of the apparatus in Fig. 23. Steam is generated in the can on

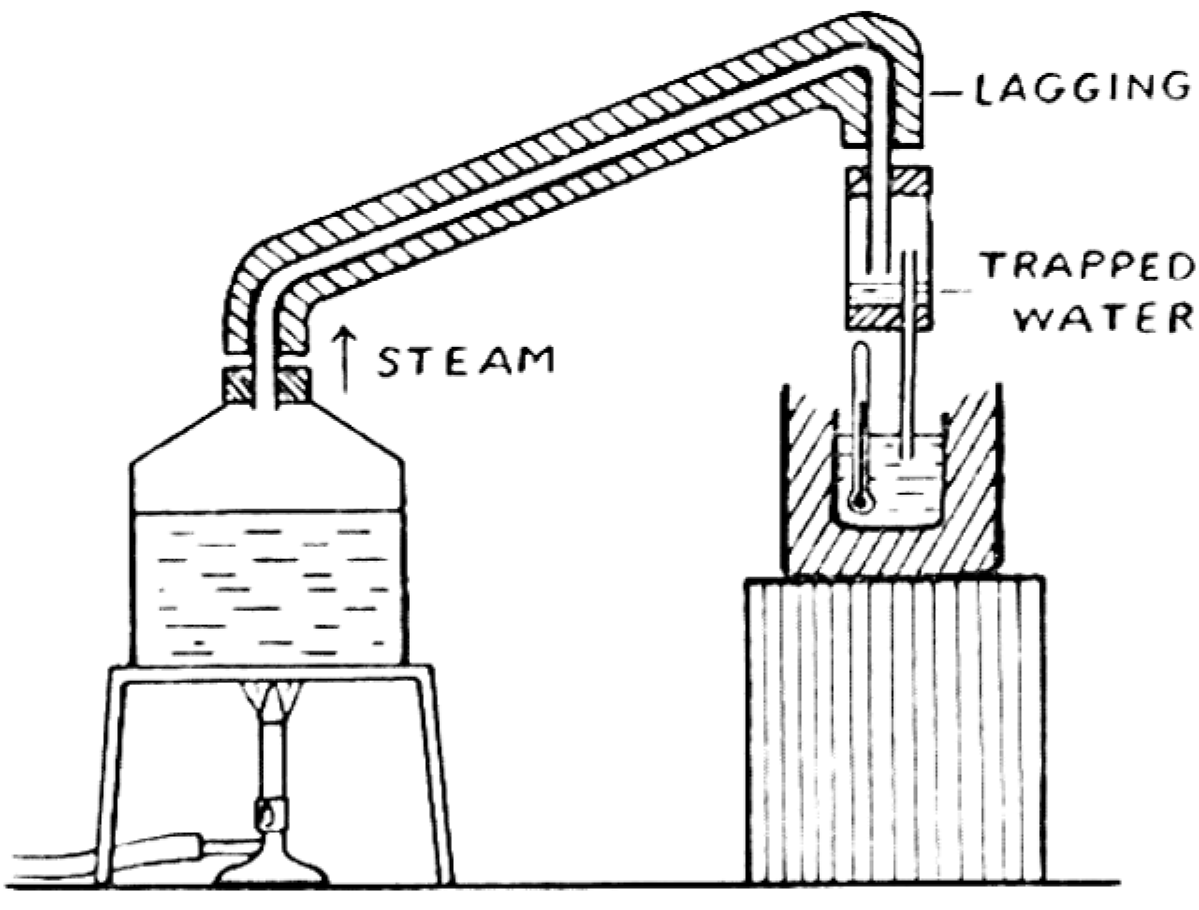


FIG. 23.—APPARATUS FOR MEASURING LATENT HEAT OF STEAM.

the left and passes through a lagged tube into a water-trap, arranged so that only dry steam passes into the calorimeter. The mass of the calorimeter, the stirrer and the water are measured and the temperature of the latter taken. Steam is then passed into the water till the temperature rises about 20° C. The maximum temperature is measured. The mass of steam condensed is found by weighing the copper calorimeter and its contents, and subtracting the previous weighing.

In carrying out this experiment, the copper calorimeter and stirrer weighed 35·60 gm. and the original water in it

51·80 gm. The temperature rise caused by passing in steam was 17° C., the initial temperature being 14° C. The mass of steam condensed was 1·55 gm.

Let L = latent heat of steam

Heat gained	=	Heat lost
by (*a*) calorimeter		by (*c*) steam condensing
(*b*) water		(*d*) water formed

$$(35{\cdot}60 \times {\cdot}095 + 51{\cdot}80)(31 - 14) = 1{\cdot}55L + 1{\cdot}55(100 - 31)$$
$$55{\cdot}2 \times 17 = 1{\cdot}55L + 1{\cdot}55 \times 69$$
$$L = 536 \text{ cal./gm.}$$

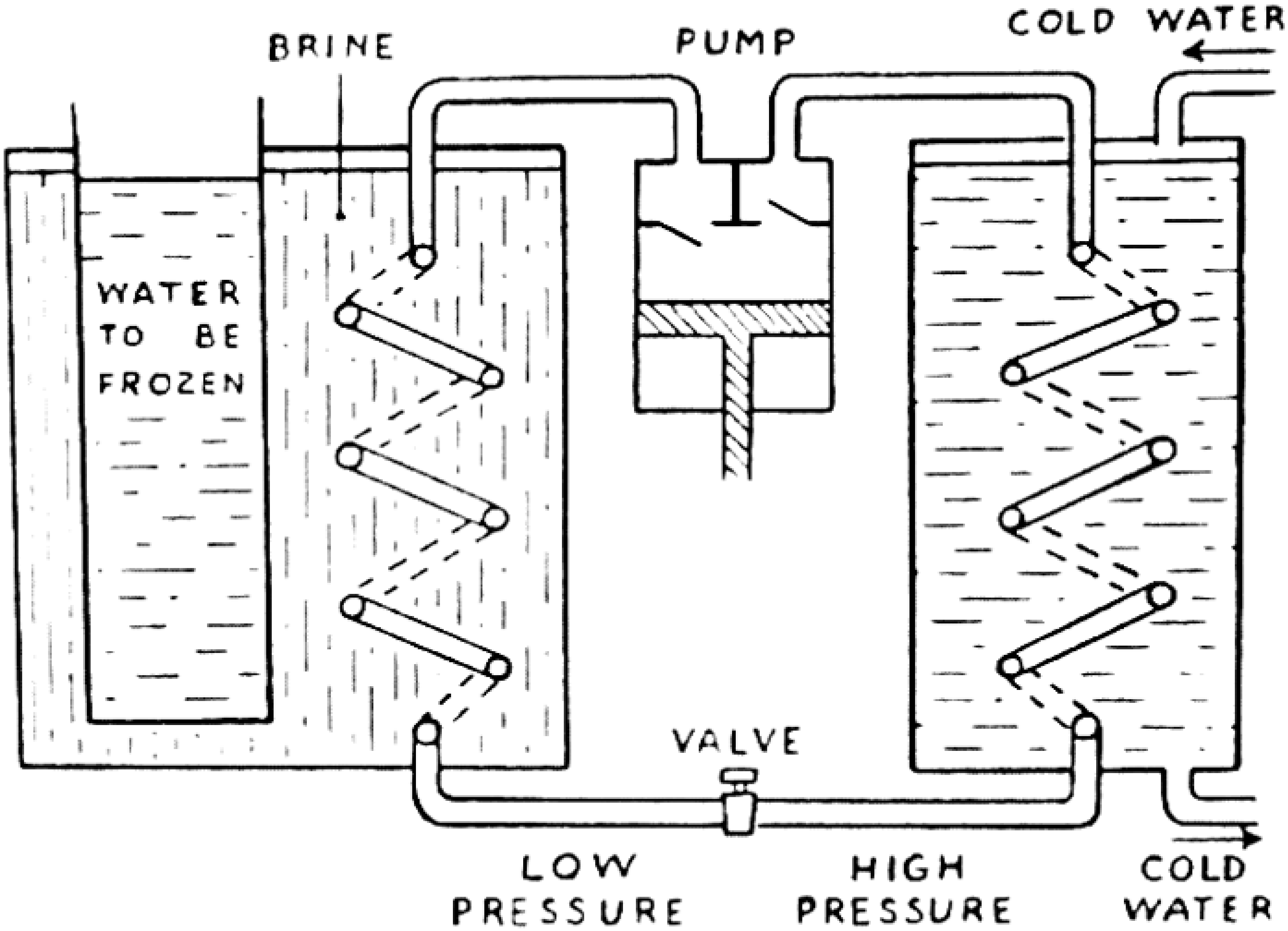


FIG. 24.—PRINCIPLE OF REFRIGERATOR.

This experiment suffers from a number of errors. We have assumed the boiling point of water to be 100° C., whereas this is only true when the barometric pressure is 760 mm. of mercury (see p. 69). Despite the water-trap, a little water enters the calorimeter along with the steam. Some of the heat given up by the steam is lost to the surroundings.

However, not all these errors have the same effect on the result, so that they partly compensate each other.

Refrigeration

An interesting application of a knowledge of latent heat is found in an ammonia refrigerating plant (Fig. 24). A steam-driven pump forces ammonia gas under pressure into the coils in the right-hand vessel, causing it to liquefy there. In liquefying, latent heat is given out, but this heat is carried away by a stream of cold water passing round the pipes. The liquid is admitted slowly by means of the valve into the left-hand coils, where it vaporises again, and obtains the latent heat to do so by chilling the brine, which can then be used for refrigeration. The vapour is drawn into the pump, and so begins the cycle again.

CHAPTER IV

PROPERTIES OF VAPOURS

Vapours

If a few drops of water are left in a saucer, they will disappear in a few hours. This occurs quite quickly on a dry day. The molecules of any liquid are in a continuous state of vibration, and since they are closely packed, collisions between molecules are very numerous. Some of the molecules gain sufficient energy in these collisions to be able to leap out of the liquid, despite the attracting force of the remainder trying to prevent them leaving. This process is going on continually in any liquid. Some of the molecules which have escaped afterwards return to the liquid. If there is a wind over the surface of the liquid, the chance of a molecule returning to the liquid will be greatly reduced, thus explaining the way in which a wet road dries quickly on a windy day. This evaporation of the liquid, as it is called, involves a reduction in the molecular energy of the liquid, and so cools the liquid. That this is so can be readily shown. Some ether is placed in a beaker standing on a piece of wood which has been soaked in water. Blow down a glass tube into the ether. The ether is agitated, its surface area increased and so its rate of evaporation becomes faster. To obtain enough energy to evaporate more quickly, the ether abstracts heat from the water through the glass, causing the water to freeze. The beaker will soon be found to be securely frozen to the wood.

If a liquid is placed in a closed vessel, after a time an equilibrium is set up in which as many molecules re-enter the liquid as leave it. Since the vessel is closed, it is impossible for molecules to leave it, and, because the space above the liquid has as many vapour molecules as it can accommodate at that temperature, the space is said to be filled with *saturated* vapour. The pressure of the vapour molecules on the walls of the vessel (i.e., the force per unit

area) under these conditions is called the saturated vapour pressure at that temperature. It is the *maximum pressure which that vapour can exert at that temperature.* As the temperature increases, so does the saturated vapour pressure, until the saturated vapour pressure becomes equal to the atmospheric pressure. The liquid is then said to be boiling, and it will be noticed that bubbles of vapour will rise from the bottom of the liquid and reach the surface before they burst. *The normal boiling point of a liquid is that temperature at which the saturated vapour pressure is 76 cm. of mercury.*

Fill a clean, stout-walled glass tube about 34 cm. long, closed at one end, with mercury. Place the thumb firmly over the open end and invert the tube, keeping the thumb in position. Now bring the open end of the tube under the surface of some mercury in a small bowl. Remove the thumb. Some of the mercury will fall out of the tube, but the height of that which remains above the level in the bowl will indicate the barometric height at the time, and the space above the mercury will be a vacuum. Proceed in the same way with two other tubes, so that three barometer tubes are standing in the same bowl. Into one of the tubes introduce a *small* drop of water by means of a pipette whose end has been bent in the form of a hook. It will be found that the level of the mercury will fall slightly, but the water will entirely disappear as it evaporates in the vacuum above the mercury. That space now contains an *unsaturated* vapour. *Such a vapour approximately obeys Boyle's law, so long as the vapour remains unsaturated.* Into another of the tubes introduce sufficient water to fill the space above the mercury with vapour and to leave some water on top of the mercury. The space above the mercury is then *saturated.* Measure the heights h_1, h_2, h_3 of the mercury columns (Fig. 25). In the case of the third tube add to the height of the mercury $\frac{1}{13 \cdot 6}$ times the height of the water layer to get the equivalent mercury height h_3'. This is because the density (or mass per unit volume) of water is $\frac{1}{13 \cdot 6}$ of that of

mercury. Then the pressure of the unsaturated vapour is $(h_1 - h_2)$, and of the saturated vapour $(h_1 - h_3')$ cm. of mercury. The latter pressure will be the greater. Now warm the tube containing the mercury and saturated vapour gently with a gas flame. The mercury level will fall, showing that *the saturated vapour pressure increases with temperature.*

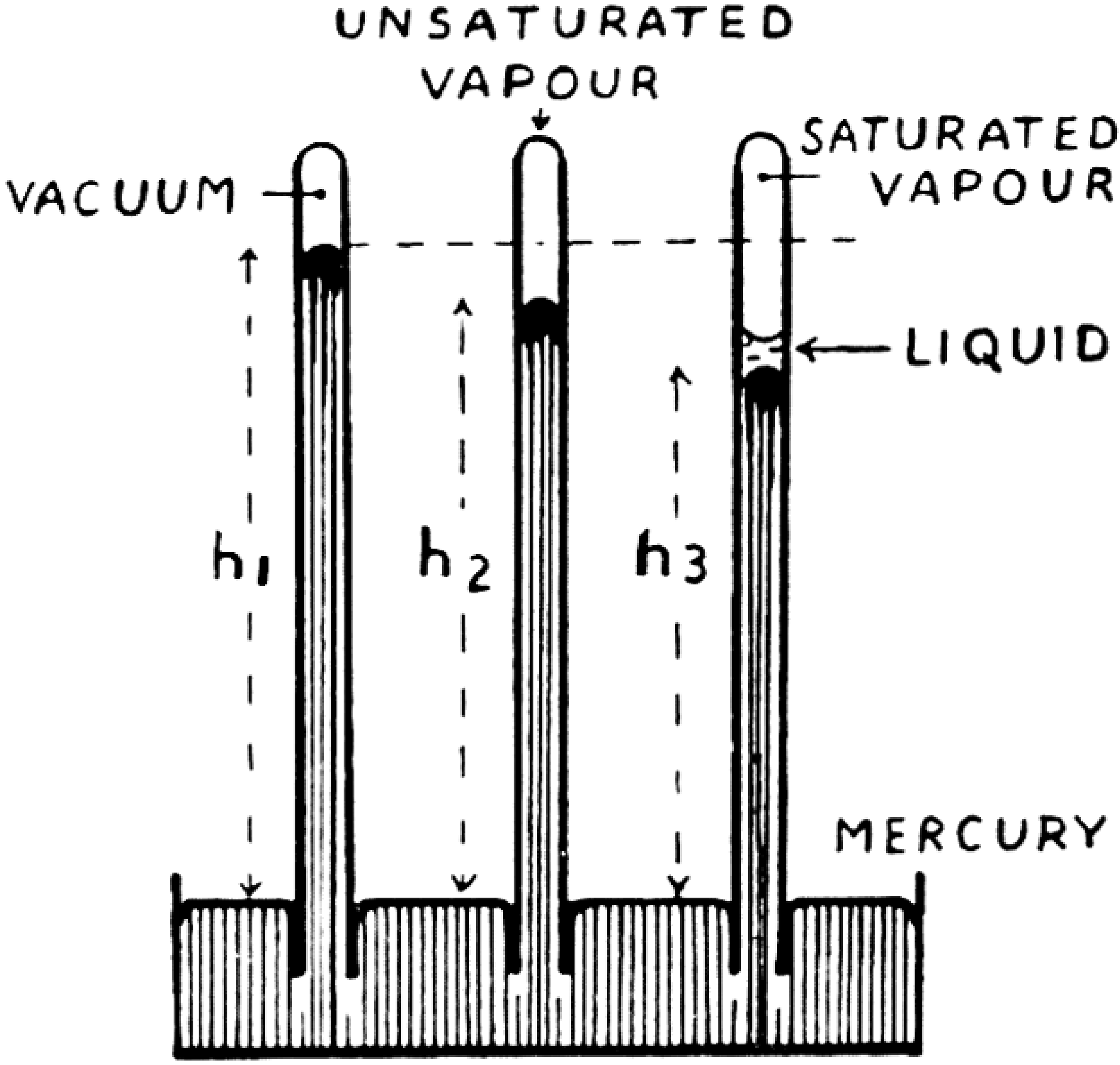


FIG. 25.—VAPOUR PRESSURE.

Now place a wider glass tube, with a cork in the upper end and a narrow tube passing through the cork, over the barometer containing the saturated vapour. Steam is then passed into the wider tube, and the temperature of the saturated water vapour in the inner tube increases until it reaches the boiling point of water. It will then be found that the mercury has been expelled from the tube, showing that *the saturated vapour pressure, at the boiling point of its own liquid, is equal to atmospheric pressure.*

Though the normal boiling point of a liquid is when its saturated vapour pressure is 76 cm. of mercury, liquids will boil when the pressure is above or below this value. Thus, to increase efficiency, steam engines often operate at high pressures. Water boiling at 200° C. exerts a saturated vapour pressure of over 15 atmospheres!

It is easy to demonstrate water boiling under reduced

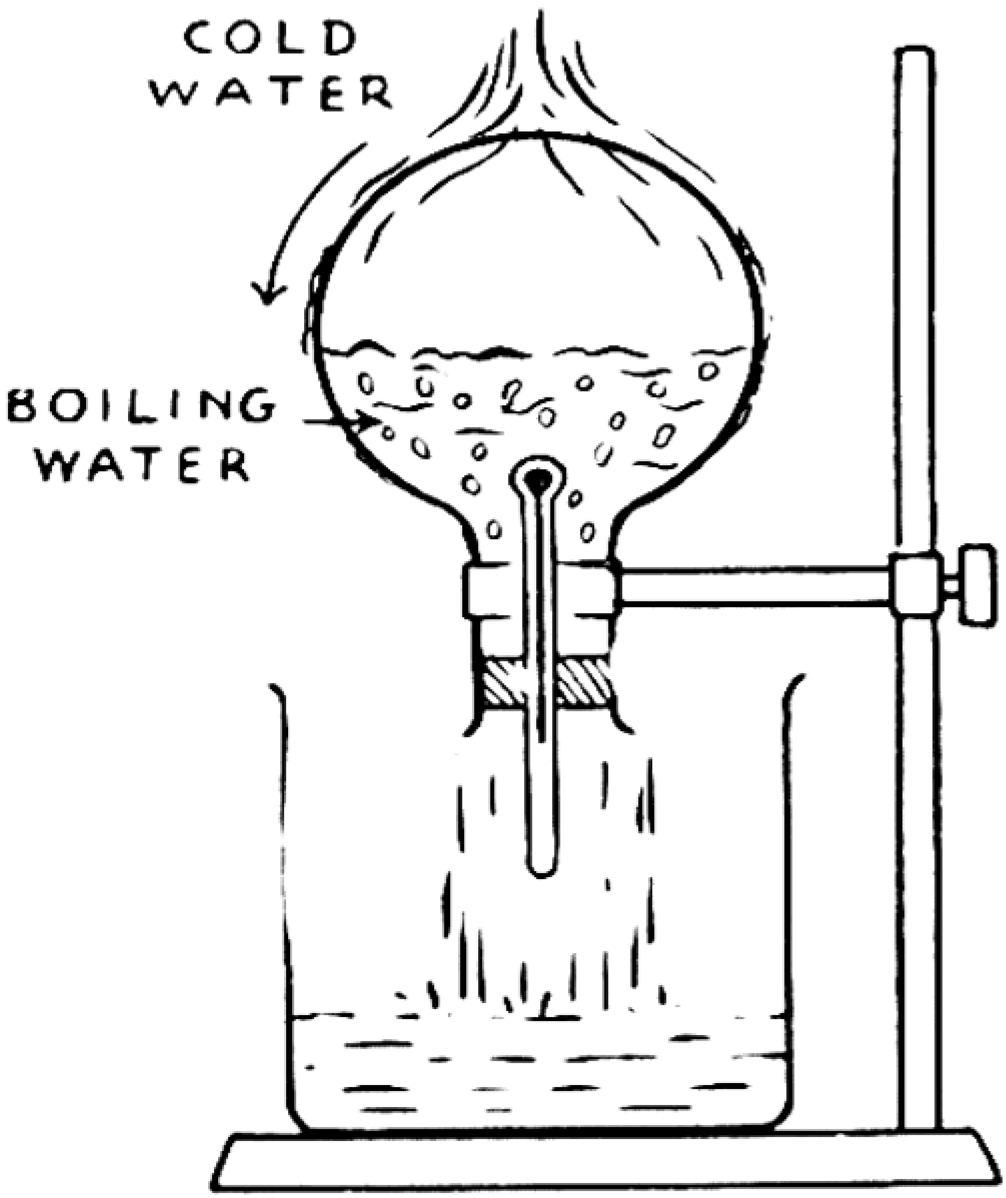


FIG. 26.—BOILING UNDER REDUCED PRESSURE.

pressure. Fit a round-bottomed flask with a tightly fitting cork, having a thermometer passing through it (Fig. 26). It is essential that there shall be no leaks in the cork. Remove the cork, half-fill the flask with water and heat it until it has been boiling for several minutes. The steam issuing takes most of the air in the flask with it, so that only water, water-vapour and a very small amount of air remain in the flask. Replace the cork and immediately remove the source of

heat from below the flask. Invert the flask and fix it in a clamp. In a short time the surface of the water inside the flask will be calm and free from bubbles. Now take a beaker containing water and pour a little of it over the flask. The water in the flask will immediately be filled with bubbles, which rise to the surface before they collapse, thus proving that the liquid is boiling. This, at first sight, is a strange phenomenon; that boiling should be caused by cooling a liquid. The cold water falling on the flask cools the steam, condensing some of it, and so lowers the pressure. This enables the water to boil again. The climbers of the Mount Everest expeditions found that at the higher altitudes it was impossible to boil an egg satisfactorily because the boiling point of water at the reduced pressure was sometimes as low as 80° C. The South American town of Quito lies at an altitude of 9000 ft. and there it is said to be impossible to make good tea, for a similar reason.

The Boiling Point of Solutions

Heat some water in a flask and note its temperature with a sensitive thermometer capable of reading to 0·2° C. Now add common salt to the water until, despite stirring, a little is left undissolved. Continue to heat the salt solution and note the boiling point. It will be found to be higher than that of the water alone. This is generally true, that *the boiling point of a solution is higher than that of the pure liquid.* A homely illustration is found in the fact that a bathing costume, after sea-bathing, will dry more quickly if it is first rinsed in fresh water, than if hung out still wet with salt water. This is because at any given temperature *salt water has a lower vapour pressure than fresh water*, and so the loss of water by evaporation, per minute, is less.

Humidity of the Atmosphere

Evaporation is continually proceeding from any water surface exposed to the air. The air molecules act as obstructions to the diffusion of the molecules of water vapour, and so delay the attainment of saturation of the

atmosphere. Warm air has a higher saturated vapour pressure than cold air and can contain more water vapour per unit volume. If warm air is cooled, a temperature will be reached when it will become saturated with water vapour, and immediately after this drops of moisture will begin appearing on any solid exposed to it.

The temperature at which the vapour present would saturate the air is called the **dew point**. It varies from day to day. If the temperature of air is well removed from the dew

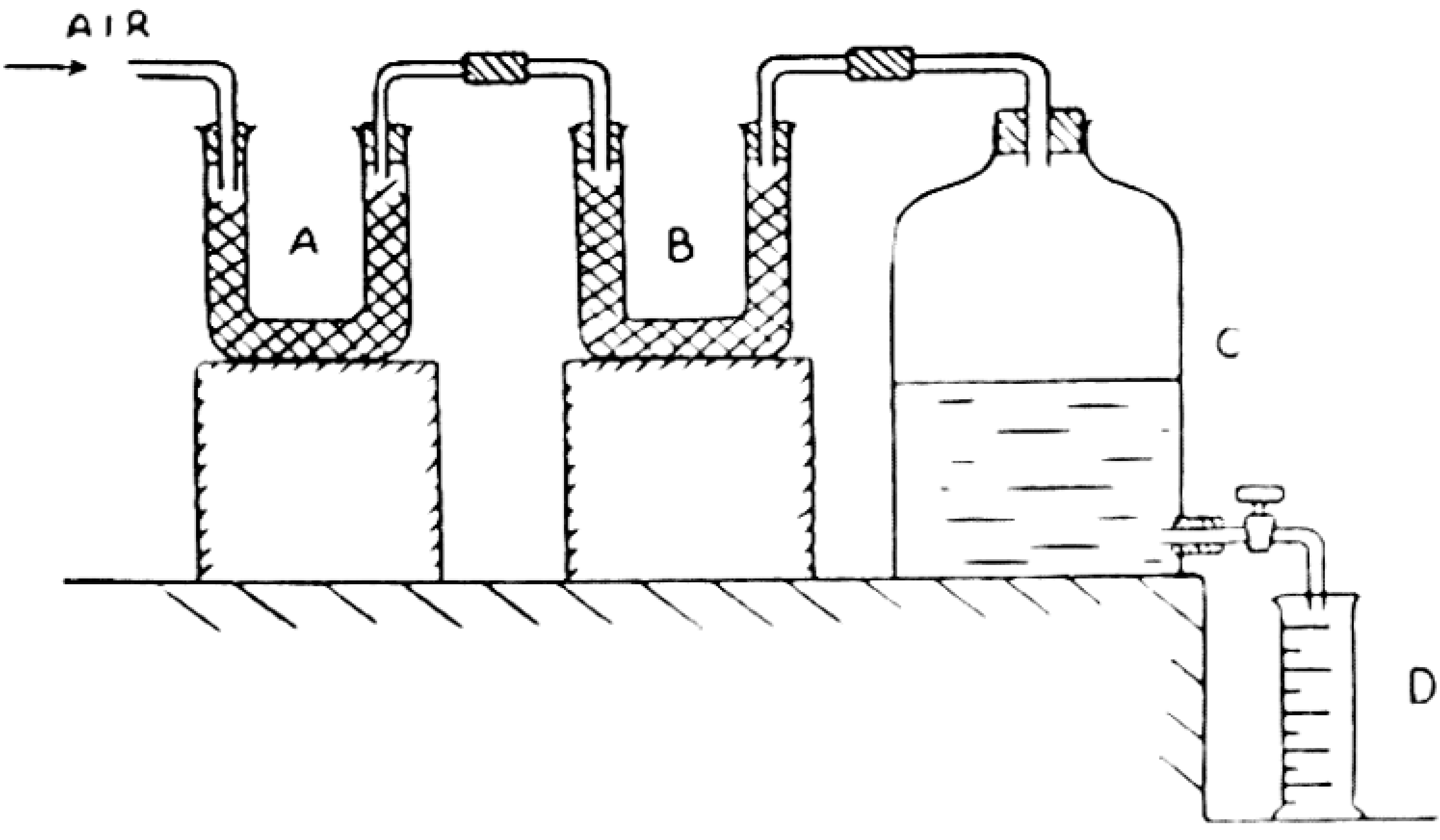


FIG. 27.—MEASUREMENT OF WATER VAPOUR IN AIR.

point, then the air feels dry, although it may contain more water vapour than cooler air which feels damp. To overcome this difficulty in defining the degree of dampness of the air, the **relative humidity** of air is defined as:

$$\frac{\text{Mass of water vapour in a given volume of air}}{\text{Mass of water vapour required to saturate the same volume of air at the same temperature.}}$$

This ratio can be determined by use of the *chemical hygrometer* (Fig. 27). The U-tubes, *A*, *B*, contain pumice soaked in strong sulphuric acid, to act as a drying agent. They are carefully weighed and then attached to the aspirator *C*, as shown in the figure. If a definite volume of water

is run out of the aspirator and measured in the graduated cylinder *D*, the same volume of air will flow through *A* and *B* into the aspirator to take its place. When a considerable quantity of air has flowed through *A* and *B*, delivering up its moisture, close the aspirator tap and re-weigh the tubes. The increase in weight will give the weight of water vapour in that volume of air. Note the temperature of the air. We have thus obtained the numerator of the above ratio. The denominator can be obtained from tables giving the weight of water vapour required to saturate a litre of air at any given temperature. Thus suppose 5 litres of air at 20° C. were drawn through the tubes, increasing their weight by ·056 gm. The weight of water vapour required to saturate 5 litres of air at 20° C. is ·086 gm.* Hence the relative humidity is $\frac{\cdot 056}{\cdot 086} = \cdot 65$ or 65 per cent. The disadvantage of this method is that the apparatus is not readily portable and that a considerable time is spent in taking a reading.

The necessity of using this method can be avoided because of the remarkable numerical similarity between the mass of water vapour in grammes in a cubic metre of air (i.e., 1000 litres) when saturated and the maximum vapour pressure which that water vapour exerts in mm. of mercury. The correspondence between the two values is fortuitous and is not exact, but is sufficiently precise for most practical purposes. Hence to find the relative humidity we can replace the ratio found above by the ratio

$$\frac{\text{Saturated vapour pressure at dew point}}{\text{Saturated vapour pressure at the air temperature}}.$$

The denominator can be found from tables, and the numerator also, provided that the dew point can be found. Hence finding relative humidity, from the point of view of the experimental work to be done, resolves itself into finding the dew point. An instrument used for determining the relative humidity of the atmosphere in this way is called a *dew-point hygrometer*.

* This information is available in tables.

A small, well-polished aluminium or copper vessel can be used to give good determinations of dew point. Half-fill the vessel with water and place in it a stirrer and an accurate thermometer, graduated to 0·2° C. A sheet of glass fixed between the observer and the vessel will prevent moisture in the breath from affecting the result. Add a small piece of ice to the water and stir steadily until the ice is melted. Put in another piece of ice and continue the process until dew is

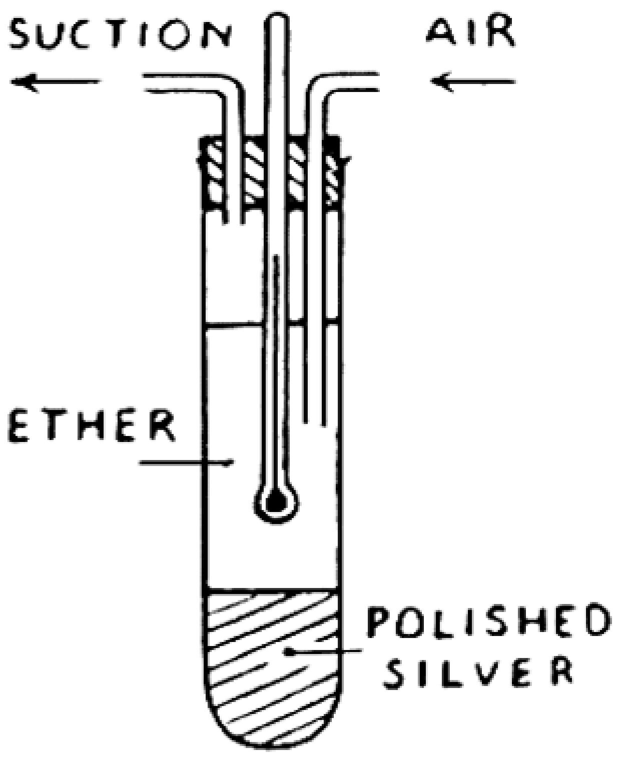


FIG. 28.—MEASUREMENT OF DEW POINT.

formed on the polished surface of the vessel. Immediately note the temperature of the water. Allow the vessel to be warmed by the surrounding air, continuing to stir. Note the temperature at which the dew disappears. The mean of the temperatures of appearance and disappearance of the dew is the dew point. In an experiment of this kind, the air temperature was 18° C. and the dew point 13° C. Hence the relative humidity was

$$\frac{\text{Saturated vapour pressure at } 13^\circ\text{ C.}}{\text{Saturated vapour pressure at } 18^\circ\text{ C.}}$$

From tables this ratio:

$$= \frac{11 \cdot 14 \text{ mms. of mercury}}{15 \cdot 33 \text{ mms. of mercury}}$$

$$= \cdot 726 = \text{approx. } \underline{73 \text{ per cent.}}$$

Regnault's Hygrometer, based on the same principle as the previous experiment, is a more reliable instrument for finding dew point. It consists of a specially made test-tube with the lower portion in the form of a silver thimble (Fig. 28). Ether is placed in the tube. Through a tightly fitting rubber stopper pass a thermometer and two tubes, one passing just through the stopper and the other nearly to the bottom of the tube. Suction is applied, by means of an aspirator, to the shorter tube, and so air is drawn into the longer tube and bubbles up through the ether, causing it to evaporate, and so cool the silver thimble. Dew is deposited on the metal surface, and so the dew point is found, as before, from the mean of the temperatures of appearance and disappearance of the dew.

Wet- and Dry-Bulb Hygrometer

This consists of two similar thermometers, A, B, suspended alongside each other. The bulb of A is wrapped in a piece of muslin whose lower end dips into a small vessel containing water. Hence water, drawn up by capillary action, begins to evaporate from the surface of the bulb of A, causing it to become colder than that of B. This can only occur if there is a draught of air round the thermometers. (If the air is still it will soon become saturated with water round A, B, and then no more evaporation will take place and the temperatures of A and B will be the same.) When used in a draught, however, the temperature of B and the difference between those of B and A are noted. With these two results, reference to tables gives the actual vapour pressure and the saturated vapour pressure at the temperature of the dry bulb B, and so the relative humidity can be obtained without knowledge of the dew point.

CHAPTER V

HOW HEAT TRAVELS

Conduction

The first mode of transmission of heat we shall consider is conduction. It is hard to find a clearer or more homely example of this than the case of a poker with one end pushed into the fire. The other end soon begins to become warm. The molecules at the hotter end have received energy from the fire and have passed some of it on to their neighbours, who have in turn shared it with their neighbours, until finally some of the heat reaches the other end. It is an

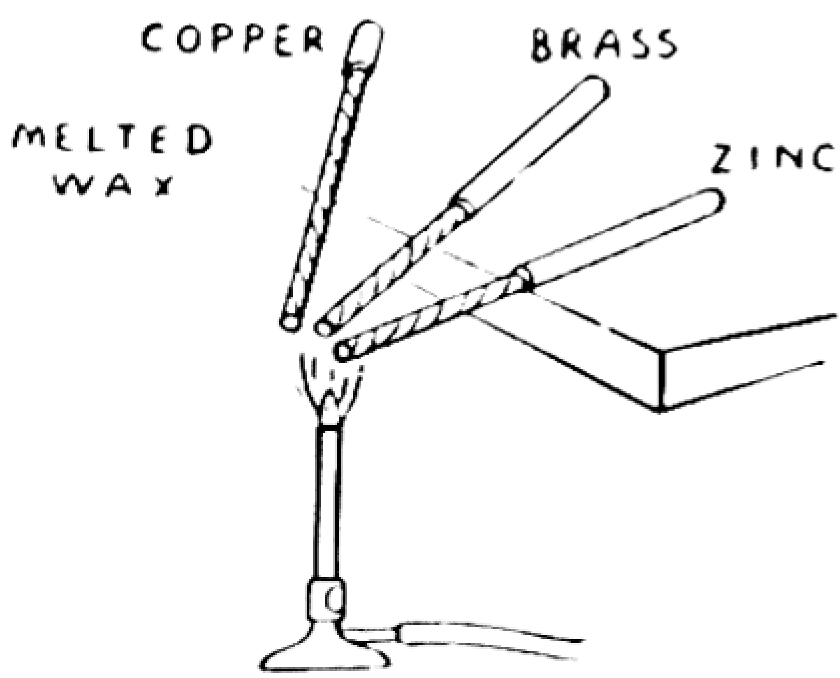


FIG. 29.—UNEQUAL CONDUCTIVITIES OF RODS.

analogous process to that of passing a shell from one soldier to another on its way to a gun, though it is energy and not a material that is being passed. In the process of conduction the particles of the conductor only vibrate about their centre of gravity, which remains fixed. Substances which transmit heat readily are said to be good conductors; those which offer resistance to the transmission of heat are called bad conductors, and those which hardly transmit heat at all are known as heat insulators.

Take three rods of copper, brass and zinc of equal length

and cross-section and dip them in paraffin wax. Allow the wax to solidify, and then place one of the ends of each together on a clay tile, the other ends being equally separated from each other (Fig. 29). The specific heats of these three metals are almost equal. Now apply a flame gently to the junction of the rods and observe the rate at which the heat travels along them from the way in which the wax melts. It will be found that the length of melted wax is greater on the copper rod than on the other two, showing that copper is the best heat conductor of the three. If the rods are long

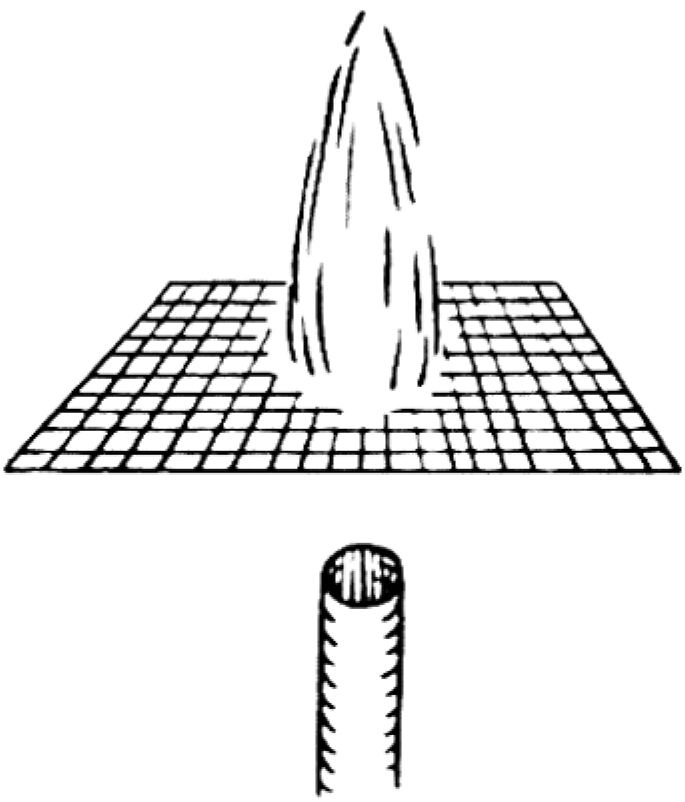


FIG. 30.—CONDUCTION OF HEAT FROM FLAME.

and thin it will be found that even on the copper rod all the wax will not be melted. At the point where the wax ceases to melt most of the heat which entered at the hot end has been lost from the surface of the rod.

The high conductivity of metals has an important application in the miner's safety lamp invented by Sir Humphry Davy. The principle of it is shown in Fig. 30. A bunsen burner is placed below a wire gauze and the gas lit above it. Not till the gas has burned long enough for the gauze to become red-hot will the flame pass through. The upper part of the gauze corresponds to the miner's lamp with the inflammable gases on the other side of the gauze. The

gauze conducts away the heat of the flame, and does not allow the gas on the other side to reach its temperature of ignition.

The Coefficient of Thermal Conductivity

Consider a block of material (Fig. 31) of cross-sectional area A, thickness d, with the two shaded, opposite faces at

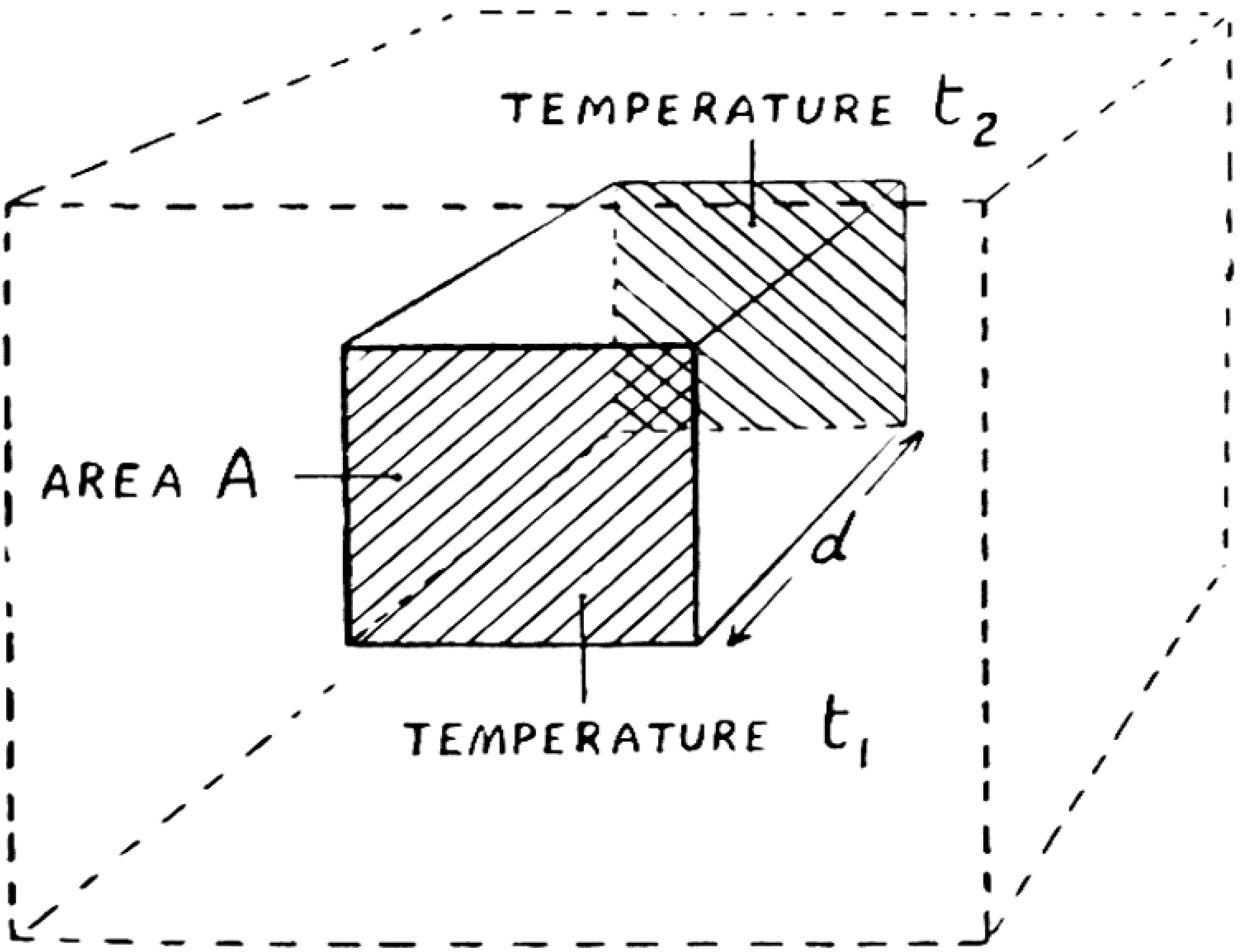


FIG. 31.—MEASUREMENT OF CONDUCTIVITY.

temperatures of t_1 and t_2, respectively, the former being the greater. The block is part of a much larger block of the same material. It is reasonable to expect (and experiment confirms) that the greater the area A the more heat Q will travel between the shaded faces, other factors being kept constant. Similarly the greater the temperature difference $(t_1 - t_2)$ the greater will be the heat flow. On the other hand, if we double the thickness d, the heat flow is halved. The greater the time T allowed, the more heat will pass

between the shaded faces. Combining all these results we can say that the heat

$$Q \text{ is proportional to } \frac{A(t_1 - t_2)T}{d}.$$

So if we introduce a constant of proportionality K we have:

$$Q = K\frac{A(t_1 - t_2)T}{d}.$$

If we take the case of unit cross-sectional area, and unit temperature difference, time and thickness, then $Q = K$ or, in words, the heat which passes between the faces will give the value of the constant K, which is called the coefficient of thermal conductivity. The value of this coefficient is:

$$K = \frac{Qd}{A(t_1 - t_2)T}.$$

The unit of d is that of length and of A is (length)^2. So the units of $\frac{d}{A}$ are those of $\frac{1}{\text{length}}$. Hence the units of the coefficient of thermal conductivity are *calories per cm. per unit difference of temperature per second*, on the metric system. What would be the units on the British system?

A detailed description of the methods usually employed to determine the conductivity of materials cannot be given here, but the following experiment illustrates the principle. Place a slab of the material to be tested, of known thickness, over a large can in which water is being boiled. It can then be assumed that the lower surface of the slab will soon reach 100° C. Measure the cross-sectional area of the base of a small, well-polished copper calorimeter and put into it a weighed quantity of water at a known temperature. Place the small calorimeter in the centre of the slab, which should be of considerably larger area than the bottom of the calorimeter. A drop of mercury placed between the calorimeter and the slab will improve the thermal contact between them. Now measure the time taken for the water in the calorimeter to rise through a definite temperature

interval. This time will be less affected by the loss of heat from the calorimeter if its curved surface is lagged.

The results from an experiment of this kind were as follows. The area of the bottom of the calorimeter, through which heat passed from the slab, was 35·2 sq. cm., and the slab was 1 cm. thick. The temperature of the water, weighing 180 gm., rose from 14° C. to 19° C. in 210 seconds. What was the thermal conductivity of the slab? At the start, the temperature difference between the opposite faces of the slab (over the area covered by the bottom of the calorimeter) would be 100 — 14 = 86° C. At the end it would be 100 — 19 = 81° C. The mean temperature difference was therefore 83·5° C. The heat conducted through the slab warmed 180 gm. of water through 5° C., and so amounted to 180 × 5 = 900 calories. (The experiment is not sufficiently accurate to justify taking the heat absorbed by the calorimeter itself into account.) Hence in the equation:

$$K = \frac{Qd}{A(t_1 - t_2)T}$$

we have

$$K = \frac{900 \times 1}{35{\cdot}2 \times 83{\cdot}5 \times 210} = 0{\cdot}0015 \text{ metric units approx.}$$

Liquids, with the exception of mercury, are bad conductors of heat. To show this for water, put a piece of ice weighted with lead into a boiling-tube nearly full of water. The ice being at the bottom of the tube, warm the upper end of the water until it begins to boil. The water is such a poor conductor that some of the ice will still remain unmelted after the water at the upper end has been boiling for a considerable time.

Gases are very bad conductors. Thus the conductivity of copper is about sixteen thousand times that of air!

Metals are often chosen for certain purposes because they are good conductors. Thus the maximum breadth of a locomotive is definitely limited by the width of the track and there are limits on its maximum length for rounding curves. Hence the surface area of the firebox is restricted

and must be used as advantageously as possible. For this reason, copper is the usual material selected, because of its high conductivity. Copper is often used for high-grade kettles. Here its advantages are partly lost by the presence of a thin *film* of gas on the outer surface of the kettle at a much lower temperature than the flame burning under it. This accounts for the possibility of boiling water in a paper vessel. This film effect is greatly reduced in large boilers by causing the burning gases to impinge on the metal with considerable velocity, thus tending to *scrub* the surface. Similarly on the inside surface of the kettle there is a thin static layer of water, which again hinders heat transmission. This is partly overcome in industrial water-heating by causing the water to move quickly, thus *scouring* away some of the static layer. The cooling fins attached to the target of an X-ray bulb are often of copper because its high conductivity assists heat dissipation. The tip of a soldering bolt is of copper to assist conduction from the main mass of the heated metal to the work.

Bad conductors have many uses for conserving and isolating heat. Thus steam pipes, boilers and refrigerators are lagged with asbestos to reduce heat transmission. Woollen clothes are excellent heat conservers. The woollen fibres themselves are poor conductors, but much more important is the extremely low conductivity of the air which lies at rest among them. The action of a haybox is similar, the hay replacing the wool.

Convection

In solids, heat can be transmitted only by vibration and collision of the heated molecules. The mean position of the molecules is fixed. In liquids and gases, however, it is possible for molecules whose heat energy has been increased to move in enormous numbers in a current or drift. This process in which *heat is transmitted by the actual movement of the molecules from place to place is called* convection.

Convection can be demonstrated in water very simply by introducing a small crystal of potassium permanganate to the bottom of a flask containing water, with the aid of a

glass tube. On heating the water round the crystal with a *small* gas flame, the water will be seen to commence circulating in the manner of Fig. 32. The crystal dissolves in the water and makes the direction of the thermal current visible. The water, heated by the flame, becomes less dense and rises, cooling as it does so. Meanwhile cold water flows down the sides of the flask to take its place. A little aluminium powder allowed to fall to the bottom of a

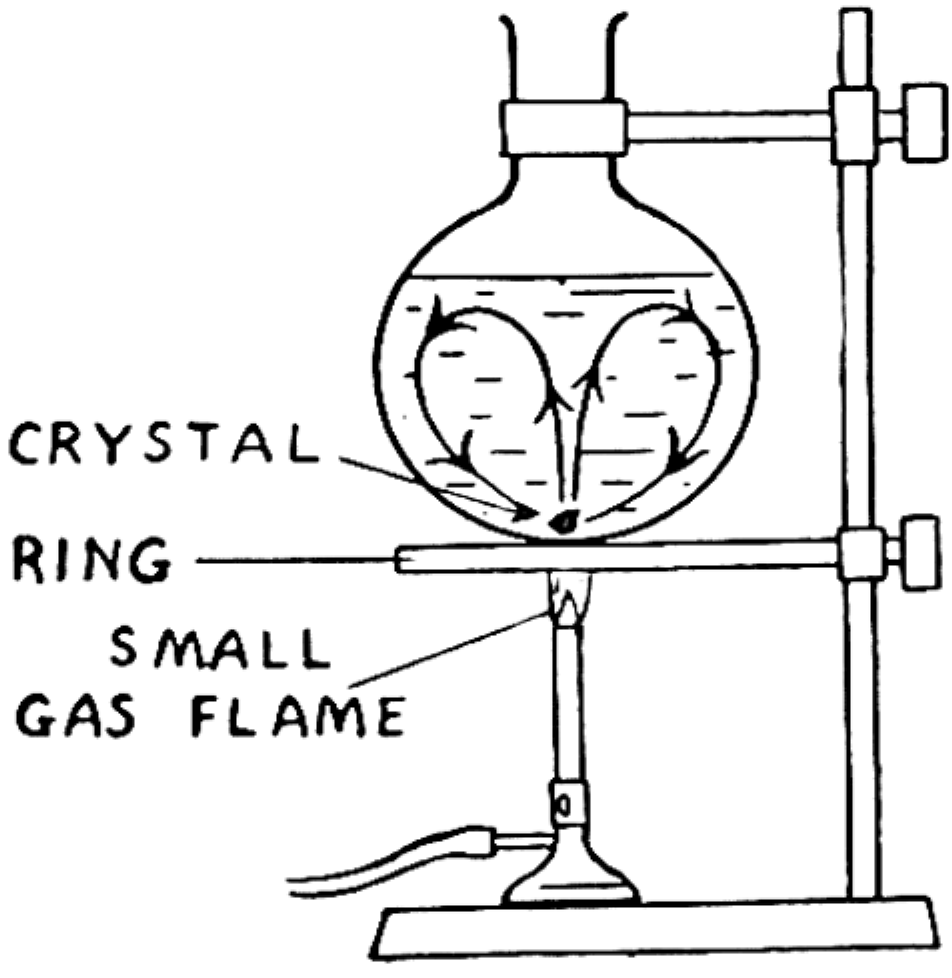


FIG. 32.—CONVECTION IN LIQUID.

flask containing benzene, which is then heated as above, gives a still better demonstration of convection currents.

Convection currents find a most important use in hot-water heating systems. A working model is shown in Fig. 33. It consists of a boiler *A* and a tank *B* connected by a tube *C* which reaches to the bottom of *A* and the bottom of *B*, and another tube *D* which connects the upper portions of *A* and *B*. A few drops of ink are mixed with the water in *B*, that in *A* being left clear. On beginning to heat the water in *A* it will rise up the flask and then up the tube *D*, thus reaching the tank *B*. As soon as this movement

begins, some of the coloured water in *B* moves down *C* into *A* to replace that which is rising up *D*. So the circulation of water from the boiler to the tank and back is established.

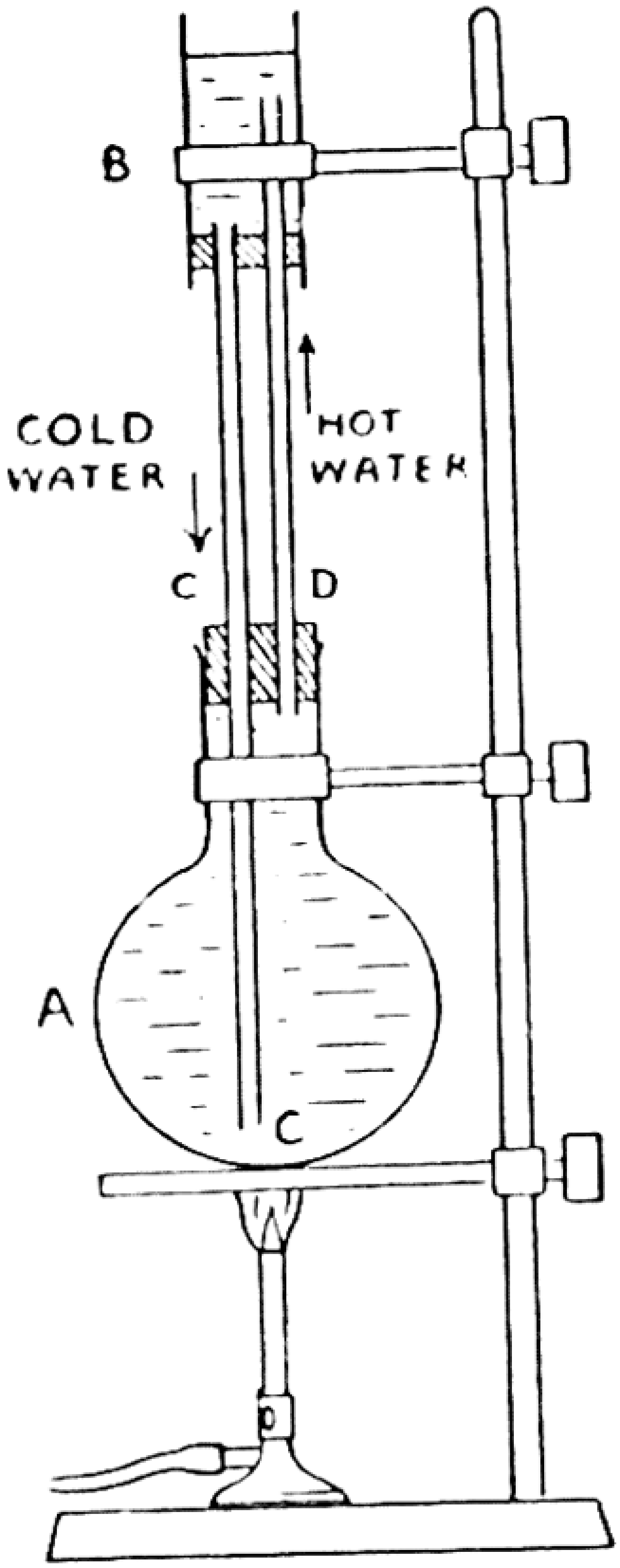


FIG. 33.—CONVECTION IN LIQUID.

In a full-scale system 'radiators' are inserted in the tube *C*. The tank *B* is at the top of the house, and the level of water in it can be kept constant by a tap connected to the cold-water main and fitted with a ball valve. For greatest pressure of circulation the ends of *C* must be as near as possible to the bottom of *A* and *B* respectively, and the ends of *D* must connect the highest portions of these vessels. This is to make the average densities in *C* and *D* respectively as widely different as possible. It is this difference which is responsible for the circulating pressure.

Good ventilation is greatly assisted by convection currents in air. In a room heated by a fire, some of the hot air rises up the chimney and some up to the ceiling. It is replaced by cool air moving in through doors and windows. As early as 1839 a house in Edinburgh was heated and ventilated by fresh air which passed over a furnace in the basement and then, rising, entered the rooms through gaps left at the bottoms of the doors. After warming each room, it passed out through louvres near the ceiling into a conduit, which discharged it just below the tiles. The rate of flow of the air could be controlled by opening or closing the louvres.

Before the invention of powerful fans, mines were ventilated by having a fire burning at the bottom of a shaft. The heated air rose up the shaft, and cool air was pushed down another shaft by atmospheric pressure to replace it (Fig. 34). A model of this system can be made by using a box to represent the workings and two metal tubes on top of it to imitate the shafts. The fire is replaced by a candle.

Air currents due to convection on a large scale take place

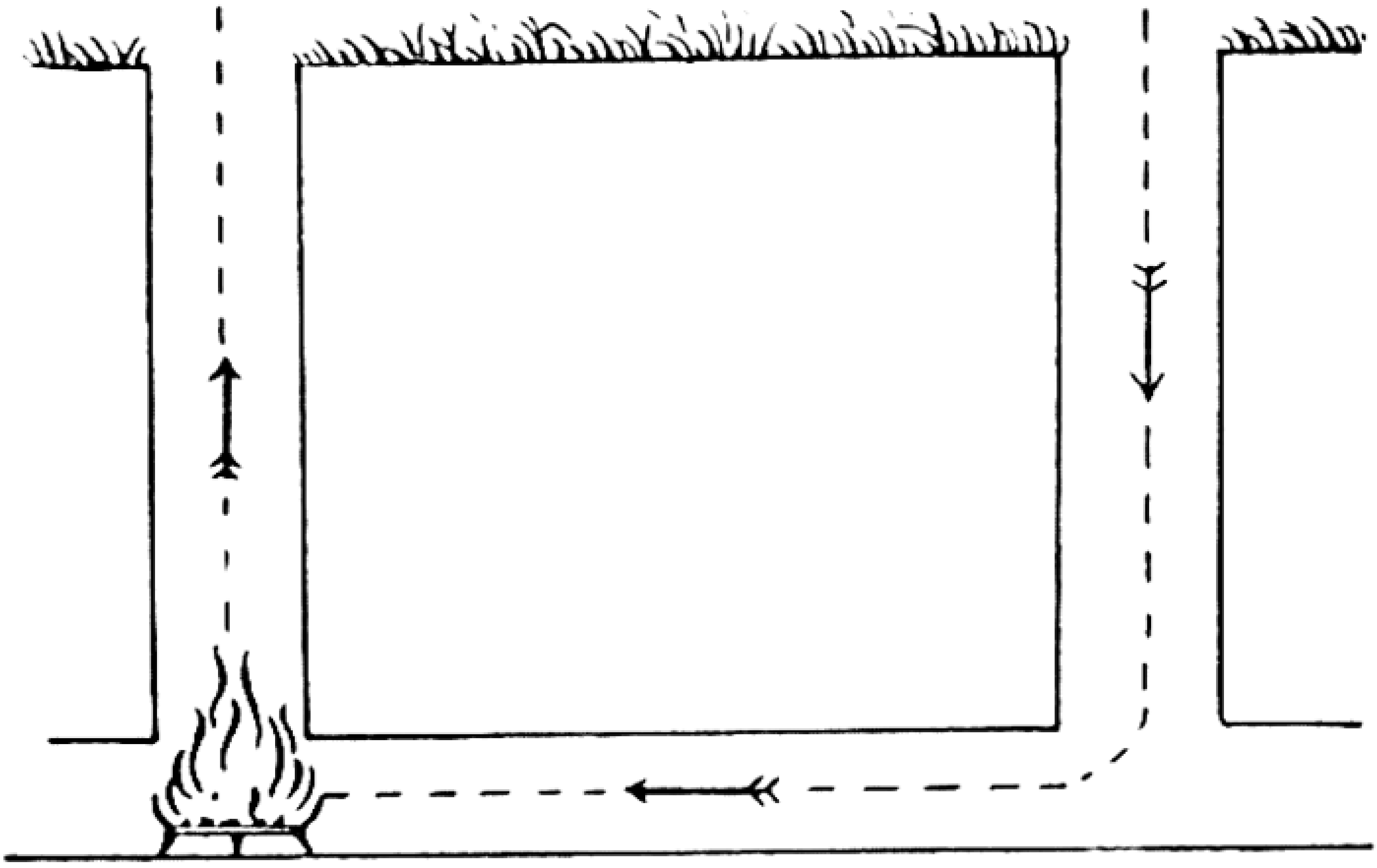


FIG. 34.—CONVECTION IN A MINE.

in the earth's atmosphere. Thus the air heated in the equatorial regions rises and air flows in from higher latitudes to take its place. The directions of these air currents would be from north and south towards the equator but for the effect of the earth's rotation, which is counter-clockwise in the northern hemisphere. If we apply an angular velocity equal and opposite to that of the earth, we shall appear to bring the earth to rest (which is how it appears to a casual observer on it). We must, however, also apply the same clockwise velocity to the wind. Hence in the northern hemisphere the direction from which the wind approaches the equator is the resultant of two velocities, from the north

and east respectively. The wind is therefore from a direction east of north. By similar reasoning that the earth's motion is clockwise to an observer in the southern hemisphere, the direction of the wind there is east of south. These currents are known as the *trade winds*.

The thermal currents of rising air set up when the sun shines down on the earth are much used by birds, and in gliding. Because of the different rates of heat absorption by, for example, cornfields and forests, the velocity of these currents varies greatly with the topography of the district, and is a cause of bumpiness in air travel.*

Radiation

I am writing this on a hot, sunny day. How does the heat reach us? Heat from the sun travels to us by radiation. It is clear, therefore, that radiated heat can travel through empty space. Physicists have never found it easy to believe that energy can be transmitted through complete emptiness, so the idea of the *aether* as an all-pervading medium has been invented. It is supposed to be responsible for the transmission of all kinds of electromagnetic energy through space, whether it be in the form of X-rays, light, heat-rays or wireless waves. Nevertheless, many careful experiments have failed to demonstrate its existence conclusively. The heat from the sun not only travels to us through space, but also does *not* heat the space. The heat which we know is only produced when the heat-rays impinge on material substances. This explains the apparent paradoxes that both airmen and mountaineers suffer from cold, although they have moved nearer to the sun. The amount of heat produced when heat-rays fall on an aeroplane or a mountain-top is small because the surface area involved is small. It is only when the waves are received on a large area such as the earth's surface that the effect becomes appreciable to us by convection from the warmed earth. The air therefore receives the sun's heat mainly at second hand from the earth's surface. The heat is radiated from

* See 'Teach Yourself Meteorology'.

the sun to the earth principally through empty space, and only for approximately 50 miles through the atmosphere. The layer of air at the earth's surface becomes heated, and then passes on its heat to the air above it to a much greater degree by convection than by radiation. (The amount of heat transmitted through any gas by conduction we have already stated to be negligible compared with that by convection and radiation.)

It is well known that frosts occur most frequently on clear, cloudless nights. This is because on cloudy nights the loss of heat from the earth by radiation is reduced by reflection back from the clouds. Radiant heat obeys the same laws as light. It can be reflected and refracted in a similar manner to light. The focusing of heat-rays by means of a burning-glass is an example of their refrangibility. Light and radiant heat are both propagated by wave-motion, but the latter is characterised by longer wave-length.

Effect of Surface on Radiation and Absorption

In the lids of two small tin containers bore a hole large enough to take a cork with a thermometer passing through it. The tins must be of equal size and shape. Remove any paper from them and paint one of them with dull lamp-black, leaving the other bright. Fill the tins with hot water from the same source. Support each tin on three corks and arrange that they are under similar conditions for cooling. Note the thermometer reading for each tin every minute. It will soon become obvious that the temperature of the tin with the dull black surface is falling more rapidly than that of the bright tin. The rate of loss of heat from the tins by convection is the same in each case, but *the tin with the dull black surface is radiating heat more rapidly than the bright surface.* This difference in the behaviour of dull matt surfaces and those which are brightly polished is true in general.

Now take another similar tin and treat its surface with paint of any colour, provided it has a dull finish when dried, and repeat the experiment with it and the lamp-blacked tin. It will be found that there is little difference between the

rates of cooling of the tins. This has an important application in the painting of the 'radiators' used for central heating. It would be distasteful to paint them dull black, but this experiment shows that almost equal efficiency can be obtained by painting them any suitable colour, provided the surface is matt. On the other hand, the experiment with the bright tin shows how wrong is the frequent practice of using bronze or aluminium paints for this purpose. To some extent, however, the reduced efficiency of radiation when using these paints is minimized by the fact that radiators give up most of their heat by convection, so that they would be better called convectors.

An electric fire of the bowl type deserves to be called a radiator because when one moves in front of the reflector the increase in heat received from it is very marked.

Empty out the water from the bright and black tins, replace it with cold water so that the tins are again both full, and place them in front of an electric fire so that they have equal opportunities of receiving heat. Observe the rate of rise of temperature of each tin. If there is any doubt that the tins have equal chances of receiving heat, reverse their positions and re-observe their rates of temperature rise. It will be found that the temperature in the blackened tin will rise more quickly than in the other, and hence *the blackened tin is the better absorber of heat.* Now, we have already observed that the blackened surface radiates heat better than one which is brightly polished. Hence it would appear (and more careful experiments confirm) that *good radiators of heat are also good absorbers.* Similarly *bad radiators of heat are bad absorbers.* A polished white surface will neither absorb nor radiate heat as well as a dull black one. It is for this reason that tropical buildings and clothes are usually white, because much of the heat falling on them is reflected back and not absorbed. Similarly many arctic animals—e.g., the polar bear, the hare and the fox—are white because their white coats lose less body heat by radiation than would darker ones. (Camouflage, it is true, is another objective in the choice of colour.)

At high temperatures (above 1000° C.) a dull black surface

radiates better than any other. An interesting experiment can be carried out with an odd piece of porcelain having a black design on a white background. If this is heated in a bunsen flame, it will at first look the same as when it was cold. When it becomes very hot, however, it appears to be reversed as a light design on a dark background. This is because at high temperatures the black design is radiating more heat and light than the white around it, and our eyes therefore receive more light from the black than from the white portion.

The *thermos flask* is an excellent example of the practical value of a knowledge of the three processes of heat transmission. It consists of a double-walled vessel, with the inner walls silvered and the air removed from the space between them. Since the vessel is of glass, the amount of heat conducted through it is negligible. Heat can only be carried by convection in a liquid or gas, so the vacuum between the walls acts as a barrier. The silvered surfaces reflect back into the vessel any heat which might have been radiated out.

Newton's Law of Cooling

Reference has already been made to this law in the description of the determination of the specific heat of a liquid by cooling (p. 57). This law stated that if a body is allowed to cool, *the loss of heat in unit time is proportional to the temperature difference between the body and its surroundings.* Actually this law is true only for small differences of temperature between the body and its surroundings. Its truth, within these limits, can be demonstrated by the following experiment.

Support a copper calorimeter on three corks inside a larger calorimeter (Fig. 35). Fill the space between them with melting ice. Inside the smaller calorimeter place a large boiling tube (or another calorimeter) fitted with a cork, having a thermometer *Y* and a stirrer passing through it. A thermometer *X* enables the temperature of the space round the boiling tube to be determined. The pur-

pose of the layer of melting ice is to keep the inner space at a constant temperature. Pour hot water into the boiling tube till it is nearly full. Replace the cork and cover the whole of the top of the apparatus with a cardboard sheet, having holes through which the thermometers X and Y pass. While the water in the tube is cooling, note the temperatures shown by X and Y every minute until the readings of Y are within 5° of those of X. The readings of X should be

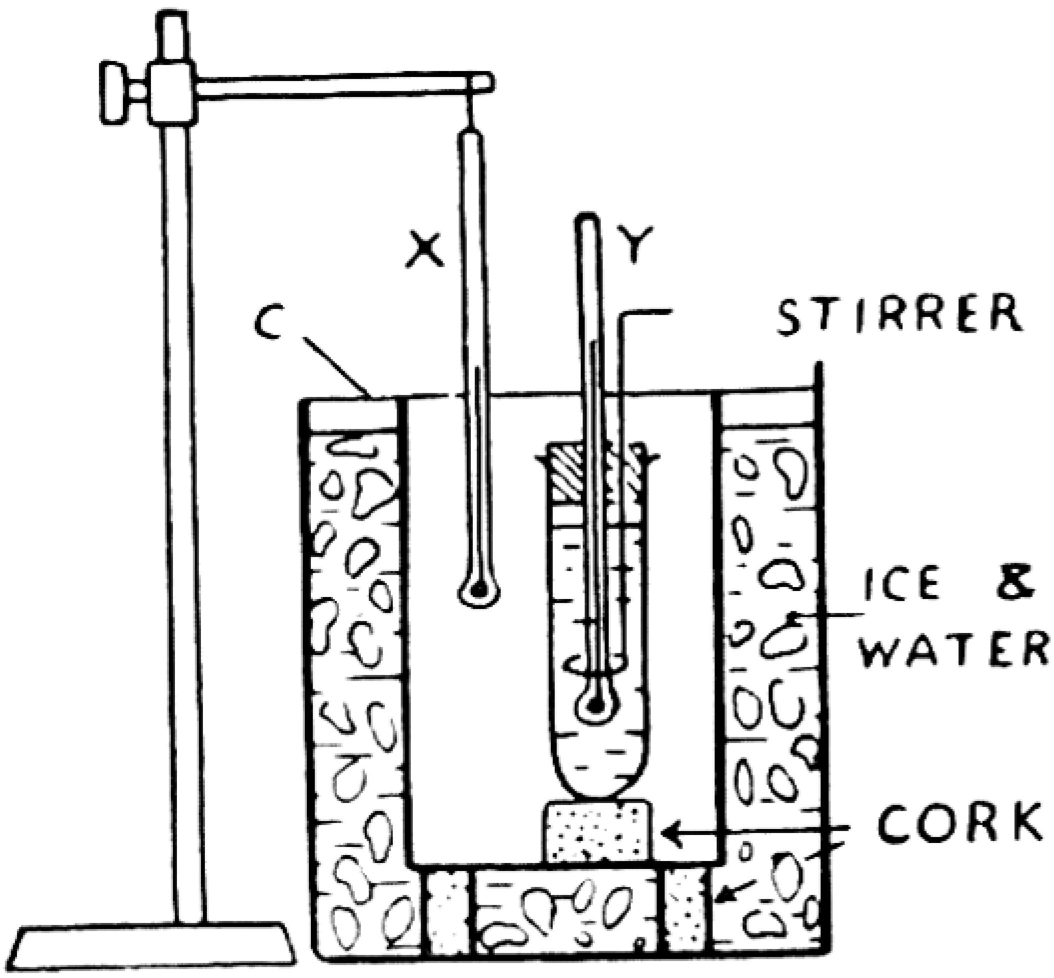


FIG. 35.—LAW OF COOLING.

constant. Draw graphs of the readings of X and Y at different times (Fig. 36). The curves LM and $ABEF$ are for X and Y respectively. Consider two equal time intervals CD, GH, separated by time DG. Then during the time CD the average temperature of the water in the tube was $\frac{AC + BD}{2}$. The temperature of the surroundings was LC throughout this time, so the average difference of temperature between the water and its surroundings was $\frac{AC + BD}{2} - LC$. The fall of temperature of the water in the same time was AK.

Hence the fall of temperature per unit difference of temperature between the water and its surroundings was AK divided by $\left(\frac{AC + BD}{2} - LC\right)$. If we consider the later, but equal interval of time GH, the value of the similar fraction would be EJ divided by $\left(\frac{EG + FH}{2} - LC\right)$. It

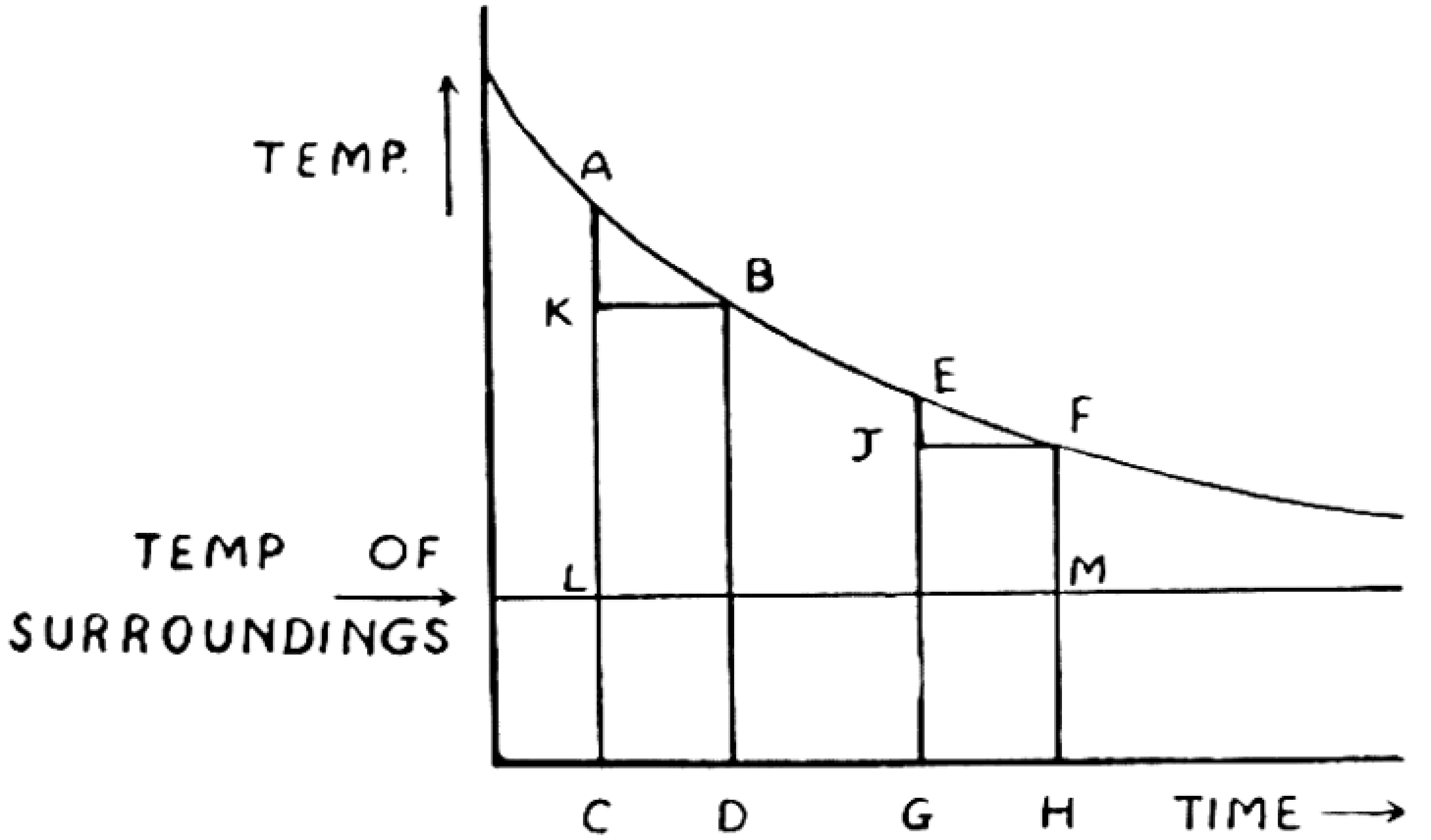


FIG. 36.—LAW OF COOLING.

will be found that the values of these fractions are equal, within the limits of experimental error, although they refer to different instants in the cooling curve. What is it that this experiment has shown? We have found that at different instants in the cooling of a body, whose temperature was not much elevated above that of its surroundings, the fraction

$$\frac{\text{Fall of temperature in a given time}}{\text{Difference of temperature between body and its surroundings}} = \text{a constant.}$$

Hence we can say that the *fall of temperature in a given time is proportional to the difference of temperature between the*

body and its surroundings. Now, the loss of heat H of a body of mass M and specific heat s when cooled by $t°$ C. is Mst calories. The mass is unchanged by temperature, and the specific heat of most substances is little affected by small changes of temperature. So in the equation $H = (Ms)t$ the factors in the bracket are constant if t is small. Consequently the loss of heat of a body is proportional to its fall of temperature. So we have demonstrated that the loss of heat of a body in a given time is proportional to the temperature difference between it and its surroundings, which is Newton's Law of Cooling.

VIBRATION AND SOUND SECTION

CHAPTER VI

VIBRATION

Characteristics of Vibration

When a body is in a position of rest and a force acts upon it, it will move off in the line of action of the force, with increasing speed. It will continue so as long as the force continues and no opposing forces are set up. Often, however, such a body is 'tied' to its position of rest, either literally or through some agency which opposes attempts to dislodge it and, once dislodged, tends to bring it back again. Such a force is known as a *restoring force*, and in many cases increases in proportion to the *displacement* which the body suffers from its position of rest. Thus, in spite of the original disturbing force, the urge to bring the body back increases until it first brings the body to rest and then overcomes the initial impulse, giving the body an acceleration in the opposite direction and returning it towards its equilibrium position. When it reaches this point, if the original disturbing force has ceased to operate, it might be expected to remain there, but, in fact, it has acquired momentum which cannot instantly be lost, so that the body usually overshoots this point and acquires a displacement on the opposite side of the zero, when again the restoring force comes into play, brings it to rest and returns it towards the equilbrium position. In general, this restoring force may be regarded as a species of *elasticity*, the acquiring and temporary retention of momentum as *inertia*. The possession of these two characteristics, elasticity and inertia, is essential for the maintenance of vibration. If the body is set in vibration in a medium in which there is considerable friction, it will not vibrate. If it is given a displacement, the restoring force operates in such a way that it is returned slowly but surely to its former position of

equilibrium. There is no overshooting of the mark because the friction prevents the body on its return journey gathering sufficient speed. Such a system is called *dead beat* and can be seen when elastic bodies which would normally vibrate readily in a vacuum or in air are immersed in a thick oil.

There are a number of such systems capable of vibration in which gravity provides the restoring force. The simple pendulum consisting of a small bob at the end of a light thread is a case in point. This normally, of course, hangs with the bob vertically below the point of attachment of the thread to the support, but when the bob is pulled aside, of necessity it rises higher than its equilibrium position, being constrained by the thread to follow the arc of a circle. The component of gravitational force urging the ball to return to its lowest position increases with its displacement, and so provides the restoring force. The momentum of the bob when it gets down swings it past this point to rise on the other side, and so on. A car at the bottom of one of the loops of a switchback railway performs similar evolutions if it is hoisted a little way up one of the inclines and let go.

Other instances can be given in which it is a liquid which performs the vibrations—still under the influence of gravity. If a stone is dropped into water, the surface is momentarily depressed and the force of gravity urges the restoration of level. The water rises out of the trough, overshoots equilibrium, forms a crest and so on.

A more obvious vibratory motion may be set up with water in a glass U-tube, if one sucks the air momentarily out of one limb so as to raise the level of the water in this limb above that in the other and then releases it. The relationship of inertia to friction to which we have just alluded may be conveniently studied in this simple apparatus. Friction or *damping of the motion* may be introduced by putting a constraint upon the movement in the form of a constriction or narrow connector either below the water level, where it impedes the water flow, or above one of the limbs, where it impedes the air circulation which has to take place when the water moves. If the constriction interposes sufficient

resistance to the motion, oscillation is inhibited and replaced by a dead-beat restoration of the water level on release of the air-suction.

Many systems can vibrate under a restoring force which is not due to gravity. This may be inherent in the system in the form of the natural elasticity of the material, or it may be due to an external force. A familiar example of the latter is afforded by the stretched ' string ', as in the violin or the archer's bow. The stretching force is applied along the length, but as soon as the string is plucked aside from the straight position or bent into an arc, components of this force are introduced which tend to restore its straightness. Systems with *internal elasticity* are found in (*a*) a steel strip clamped at one end to the bench while the other is bent downwards and let go, and (*b*) the ' spiral spring ' formed by wrapping a stiff wire round a pencil. This type of spring may have a weight hung on its lower end while the upper is clamped to a support. The weight jigs up and down when it is given a vertical displacement. Thus the restoring force is in the main due to the elasticity of the spring, while the attached mass controls the inertia.

This elastic type of vibration is not confined to solid bodies. Cavities containing air may vibrate if the air is compressed and then let go—a good deal of the sound of the ' babbling brook ' is due to this type of vibration set up in air bubbles entrapped in the stream, while, on a larger scale, we find the sound of drums due in the main to the large volume of air which they enclose being set in vibration by taps on the skin.

Characteristics of Periodic Motion

Provided the initial displacement imparted to the bodies we have described is not too great, the vibrations which they execute all partake of the same important characteristics.

(1) The motion is *periodic* in the sense that if one measures the time taken to execute a complete swing—from one extreme across to the other side and back again—this remains *constant* as long as the vibration persists. This time for a complete swing is known as the time-period (measured in

seconds). The number of complete swings executed in unit time (one second) is called the frequency of the vibration.

(2) The extent of swing decreases due to friction as time goes on, unless it is maintained by an external supply of energy. The maximum displacement of the system from its point of equilibrium as it swings is known as the amplitude, and the decay of amplitude in a non-maintained system is measured by the decrement. The decrement is usually expressed as the logarithm of successive amplitudes.

(3) *In the absence of friction, the total energy of the system during oscillation is constant.* This may be illustrated by reference to the pendulum or the ball on the switchback railway. When the ball is at the top of its orbit, it is momentarily at rest and its energy is all *potential.* At the centre of its swing it has reached its lowest point and it has lost potential energy (represented by its weight times the vertical height it has fallen through). At this point it has acquired its maximum velocity and possesses *kinetic energy* equal to half the product of its mass and the square of the acquired velocity.* It may readily be shown that these quantities are equal—in fact, that at any point in its path, the kinetic energy it has gained in virtue of its acquired speed is equal to the potential energy it has lost since it was at the point of maximum displacement.

With friction this is no longer true. As the ball descends, some of its original potential energy is converted into work that has to be done in overcoming friction. As a result of this, the kinetic energy as it passes its mid-point of swing is somewhat less than that corresponding to the potential energy lost, and the ball rises on the other side to a lower level on the railway. This decrease of height is accompanied by a decrease of amplitude, and so causes the motion to be damped unless it is maintained by an external supply of energy.

The type of simple to-and-fro oscillation that we have been discussing is known as simple harmonic motion (S.H.M.) It is a common characteristic of a system set in

* See 'Teach Yourself Mechanics', p. 184.

feeble vibration with a *small amplitude*. If the amplitude is too great, or if too much forcing is used (which comes to the same thing, in practice), two deviations from the laws of simple harmonic motion, as above described, may follow:

(1) The motion is no longer exactly periodic, the time of swing decreases as the amplitude decreases.

(2) Even ignoring the varying time-period, the vibration is not a pure to-and-fro vibration (like that of a child's swing) but is complex, in the sense that the system may perform partial vibrations (of higher frequency) at the same time, so that, for example, in between each main swing of large amplitude there may be subsidiary ones of smaller amplitude.

Graphical Representation of Vibration

If we wanted to study the features of the vibration of a pendulum, we could attach an inky feather to the bob and move a piece of paper along at right angles to the direction of motion of the bob. If we move our paper at constant speed, we obtain thus a curve in which displacement is plotted on one axis against time on the other. Fig. 37 shows some typical traces obtained in this way: (*a*) and (*b*) when the amplitude is maintained as in a clock pendulum, (*b*) having twice the frequency of (*a*); (*c*) and (*d*) in which the S.H.M. of the first trace is overlaid by a partial vibration of double its frequency.

Velocity and Energy of a Particle in Vibration

The above graphs represent displacement (of the particle) against time. Since velocity is rate of change of displacement, the velocity of the vibrating particle at any instant is represented by the slope of this curve at the corresponding point (see p. 19). This is zero when the particle is at its greatest excursion, and a maximum as it passes through its equilibrium position. By drawing a number of S.H.M.'s corresponding to Fig. 37, the reader may verify that *as the amplitude increases, so does* the maximum slope, and there-

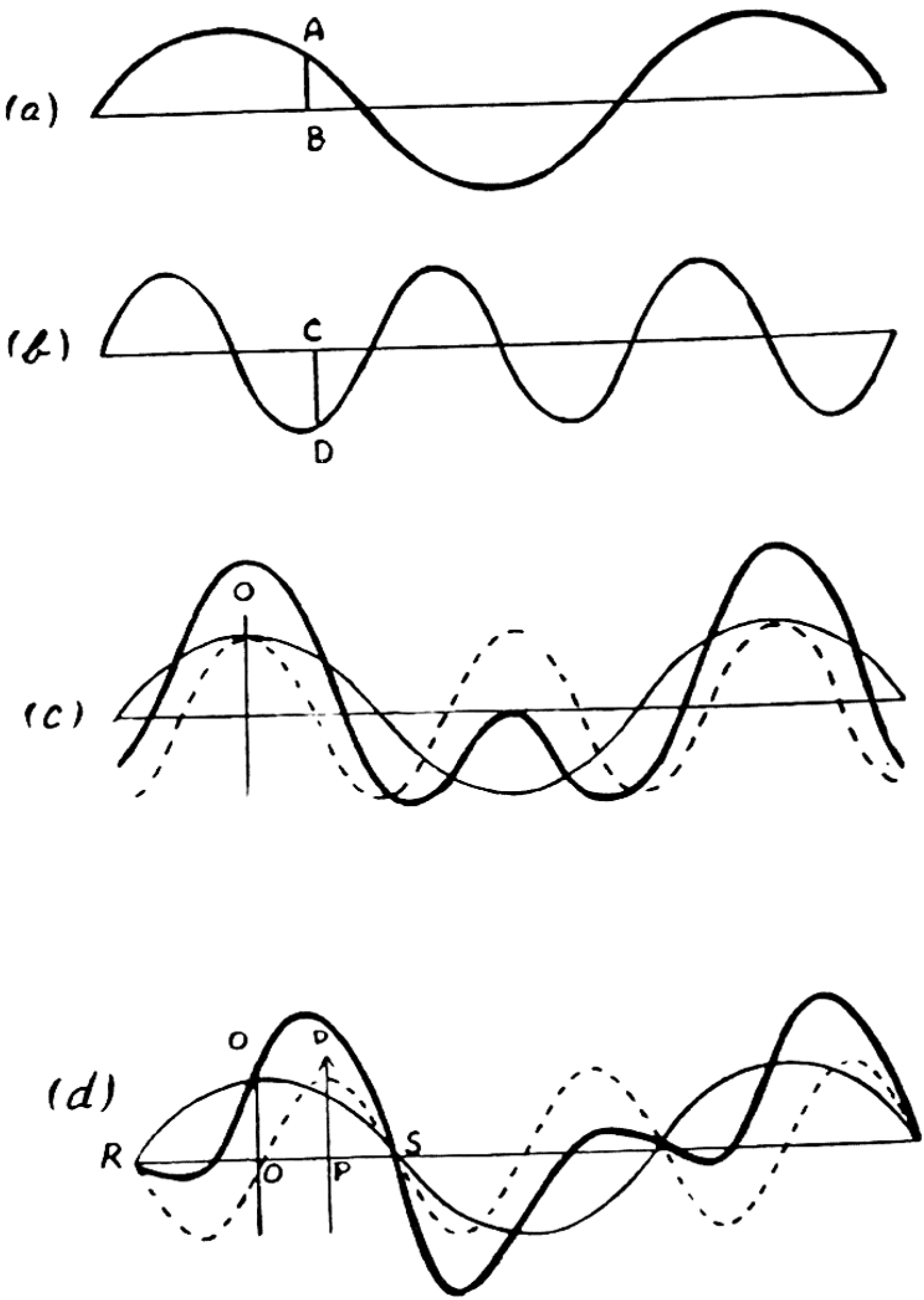


FIG. 37.—SIMPLE HARMONIC MOTION, CURVES.

fore *the velocity.* By drawing others at constant amplitude but different frequency, he may verify that the maximum velocity is proportional to the frequency.

The velocity of a particle executing S.H.M. is proportional to the product: frequency × amplitude. The kinetic energy is proportional to the square of the velocity and therefore to (frequency × amplitude)^2.

THE PRINCIPLE OF SUPERPOSITION

The Meaning of Phase

It will be noticed that the traces presented in (*c*) and (*d*) are not identical, though in each case the drawing has been done by superposing the two traces shown in (*a*) and (*b*). This superposition is done by supposing that if a body were acted on by forces which, acting severally, would produce displacements whose value is shown by the heights AB and CD, then when both forces act at once, *the displacement* at the corresponding instant *is equal to the algebraic sum of the two separate displacements.* If AB and CD are both on the same side of the mean position we add them, if opposite we take their difference. In arriving at trace (*c*) we have done this by selecting for the disturbance at the time O that at which each partial vibration would, acting alone, produce the maximum displacement of the bob on the same side of its mean position. Such vibrations are said to be in phase at O. In producing trace (*d*), however, we have set our component of higher frequency at O, where it, acting alone, would make the bob pass through its mean position, while the component of lower frequency is at maximum amplitude. These two partials are said to be out of phase at O.

Representation of Phase Difference

1. *In Terms of the Time-Period.*—In Fig. 37 (*d*) we can represent the amount that the partial of higher frequency lags behind the other by the fraction of the time-period represented by the ratio of the time OP (in seconds) to the period of either partial—OP representing the time that elapses between each component reaching its maximum.

Usually, it is the lower (or lowest) frequency which is selected as standard so that, on this reckoning, the difference of phase between the two components is represented by the ratio $\frac{OP}{RS}$, or one-quarter of the time period.

2. *As an Angle.*—Imagine a ball A going round a circle (Fig. 38) at constant speed, and another ball B constrained

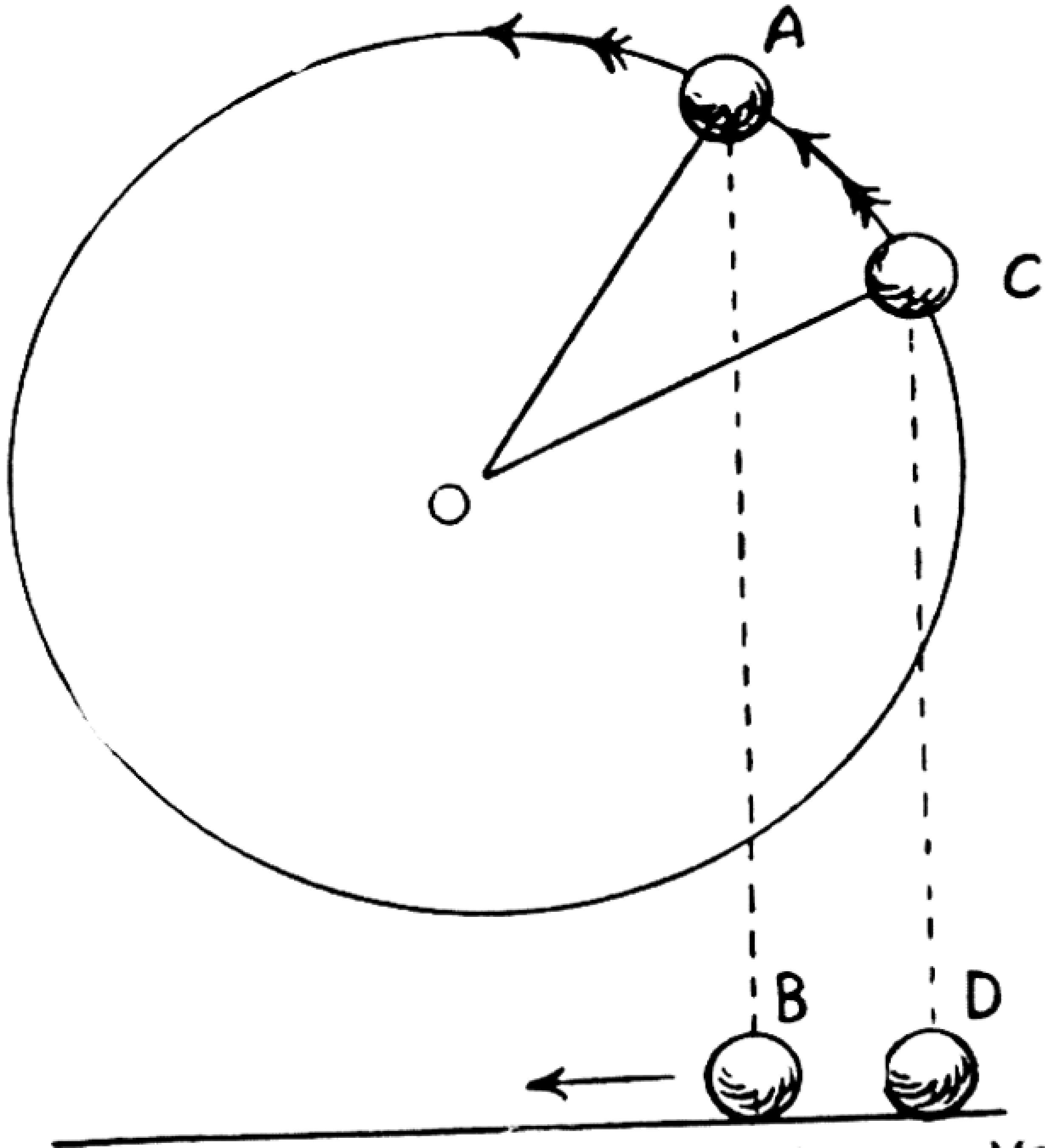


FIG. 38.—SIMPLE HARMONIC MOTION AND CIRCULAR MOTION.

to move in a straight groove, of length equal to the diameter of the circle in such a manner that B always 'covers' A (in the military sense). (If the book be held up so that the page is vertical, B is to lie always vertically beneath A.) Then while *A executes circular motion at constant velocity, B executes simple harmonic motion.* This relationship can

also be exemplified by whirling the bob of a pendulum round a circle in a horizontal plane and looking at it first from above and then with the bob on a level with the eye. The amplitude of B's S.H.M. = the radius of the circle. The time-period of B's S.H.M. = the time of one revolution of A in the circle. The frequency of B's S.H.M. = the number of revs. of A per sec.

Let another ball C move round the circle with the same speed as A but always at a fixed distance behind. Let D in the groove 'cover' C in the circle. Then D executes S.H.M. of the same amplitude and frequency as B, but with a phase lag, which is represented by the angle AOC (measured in radians).

If we increase $\angle AOC$ to a right angle, the phase lag is $\frac{\pi}{2}$.

If we increase $\angle AOC$ to two right angles, the phase lag is π.

In the latter case, the vibrations of B and D are in 'dead opposition' of phase—that is, one is at its left extremity of excursion, while the other is at its right.

If $\angle AOC$ is increased until C comes round to coincide with A, the phase difference is 2π (or zero) and the motions are in phase again.

Interference; Beats

When two sets of vibrations of different frequencies act simultaneously on the same system, then the principle of superposition tells us that the displacement of the system at any instant is the algebraic sum of the displacements which either would, acting alone, produce at that instant. At certain times, due to the two vibrations acting in opposition, the resultant displacement will be less than that due to either alone and will be zero if the two vibrations demand simultaneously equal and opposite displacements. This is a case of quite common occurrence, and is known as interference. It may often be seen where two sets of ripples meet on the surface of a pond.

In Fig. 37 (c) and (d) we have depicted the superposition of two vibrations having frequencies in the ratio 2 : 1.

When the frequencies have no common denominator and are closer together, an interesting state of affairs ensues. Fig. 39 shows at (*a*) and (*b*) the displacement: time curves for two vibrations having frequencies in the ratio 10:9 (viz. (*a*) completes 10 half-periods while (*b*) completes 9). The resultant obtained by summing them is shown at (*c*). The vibrations are in step at the start: the amplitude is the sum of those of the two components. After 5 vibrations of

a

b

c

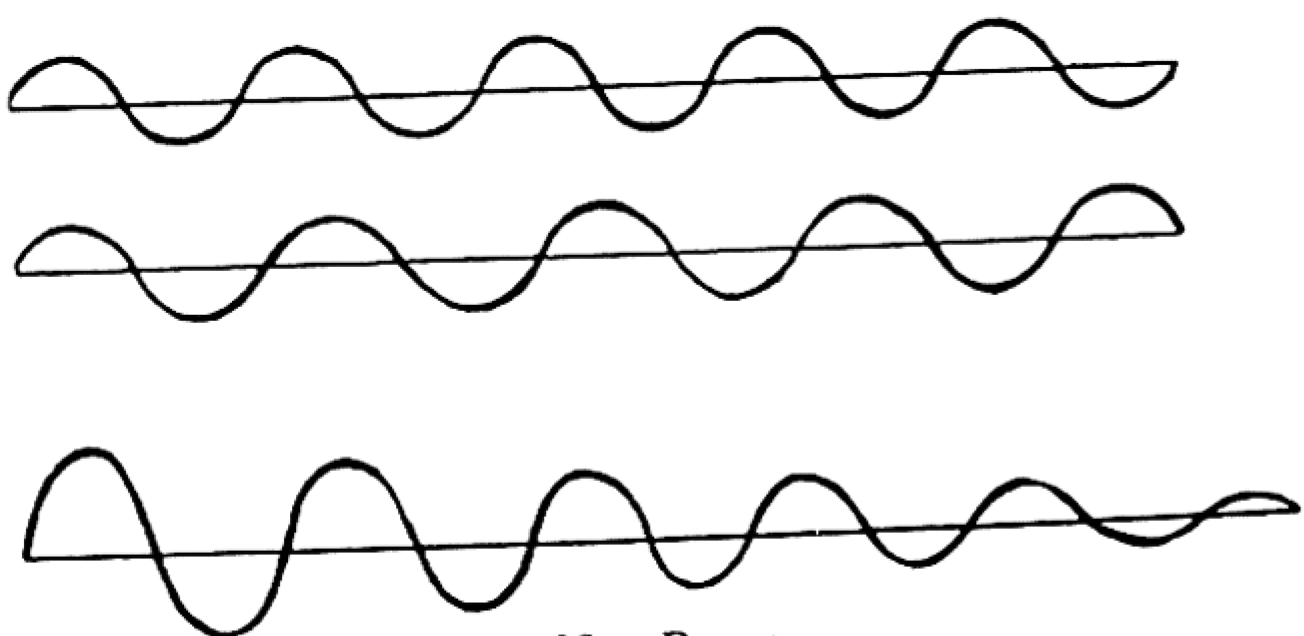


FIG. 39.—BEATS.

(*a*) and 4½ of (*b*) they are out of step. The amplitude at this point is the difference of those of the components (in the case depicted—zero). After 5 more of (*a*) they will be in step again. In between, vibration has continued at a frequency approximately equal to that of (*a*), but with amplitude first falling and then growing. This waxing and waning will continue at a *frequency equal to the difference between those of* (a) *and* (b) *alone*. These slow changes of amplitude, superposed on the faster vibrations of the components, may sometimes be seen when a cork on the surface of a pond has two sets of waves of different frequency passing over it. They are known as beats and are important in sound.

CHAPTER VII

WAVE MOTION

Origin of Waves

A wave occurs whenever we *displace one end or one point* of a connected system of elastic particles, as opposed to giving the whole system displacement simultaneously over all its parts. The fact that a wave is propagated along the system is due to its having some form of elastic connection between its particles. If a line of wagons on a railway track is so rigidly coupled that no motion of one truck relative to its neighbours is possible, then an impulse communicated to the end truck must result in the simultaneous movement of the whole train, but if they are loosely coupled, an impulse communicated to the one end will be passed along the train with a time lag between the motion of consecutive trucks, so that one can watch the buffer springs contracting and relaxing in turn as the impulse is passed along. The impulse is said to be *propagated with a velocity* which can be measured by noting with a stop-watch how long the pulse (as such a single disturbance is called) takes to pass from the second to the twelfth truck in line, for example.

The impulse need not be directed along the line. In the case of a rope stretched between two posts one can cause a pulse to travel along by smartly plucking the rope aside at a point near one end.

If in place of a single ' pluck ', one agitates the end of the rope to and fro in, roughly, simple harmonic motion, a *continuous wave* travels along the rope. Each part of the rope takes up the motion a little later than its predecessor, but each with the *same frequency* and the *same amplitude* (provided friction is not paramount). The phase lag increases progressively along the rope from the point of agitation, so that points farther away are doing their evolutions progressively later behind the first point moved, until we come to a point which lags 2π behind number one—namely, is in step with it again, and so on.

The resulting picture of the wave imagined to be photographed at any instant is precisely like that of Fig. 37*a*, when in place of time along the horizontal axis we must read distance. At a later instant the whole picture will have moved to the right (if that is the direction in which the wave is moving) by a distance equal to the product of the elapsed time and the velocity of the wave. The distance between two points on the 'photograph' which are moving in phase is called the wave-length; alternatively it may be defined as the 'distance between two crests' in the distance over which the wave pattern repeats itself.

If the end of the rope is given a S.H.M. of definite frequency (vibrations per second), then that crest which was initially at the near end of the rope will, after the elapse of one second, be at a distance away equal to the rate of travel of a crest, divided by the length between crests; or, the velocity of the wave divided by the wave-length.

Thus in a progressive wave,

$$\text{Velocity of wave} = \text{frequency} \times \text{wave-length.}$$

It should be borne in mind that each part of the rope moves to and fro only, and that the rope itself does not bodily move along. It is only an appearance which moves along the rope, as each particle goes to a crest in turn.

Types of Waves

We have already mentioned two types of progressive wave; (*a*) that exhibited by shunting a train of wagons is called a longitudinal wave, because the *displacement* of each part *is along the direction of propagation* of the wave; (*b*) that exhibited on the rope of which one end is plucked aside is called a transverse wave, because the *displacement* of each part is *across the direction of propagation* of the wave. There is a third type, less common, (*c*), called a torsional wave, in which the initial disturbance is a twist of part of the system.

All these partake of the same characteristics as to amplitude, frequency and velocity, though, for convenience, the

diagrams which we draw to illustrate wave motion are usually of the transverse type.

Reflected Wave

It will be noted, in the case of the rope, that when a pulse of disturbance reaches the far end it is *reflected*—that is, a wave comes back to the sender along the rope as it reaches the far end. Reflection of waves is, of course, quite common in nature. It may be seen when ripples formed on a pond hit the bank.

The phase of the reflected wave where it leaves the boundary may bear any relation from zero to π (half a period difference) to the oncoming wave at the instant of reflection, but two important cases arise:

(1) When the end is perfectly free and without constraint, the *reflected wave is in phase with the incident.*

(2) When the end is fixed and perfectly constrained, the *reflected wave is in opposition of phase with the incident.*

Thus at a free end a crest on arrival is reflected as a crest; at a fixed end a crest sends back a trough.

Stationary or Standing Waves

When a periodic series of oscillations is sent along a system and reflected at each end interference occurs, and if the timing of the oscillations be correct stationary waves are set up. These are so called because there is no appearance which moves along, merely a variation in amplitude from place to place, having permanently still points spaced at equal distances (these are called nodes or knots) with—half-way between—places of maximum amplitude called antinodes or loops. Between the nodes and antinodes the amplitude varies from zero to a maximum, but all points move in step, seeming rather like the motion of a skipping-rope viewed from one side. When there are several segments, alternate loops are out of phase, so that one is up while the next is down. The wave-length is still equal to the 'distance over which the pattern repeats itself', in this case two loops, from one node to the next but one in line.

The way in which stationary waves are built up from two equal progressive waves is illustrated in Fig. 40, where we have drawn four successive stages in two such progressive

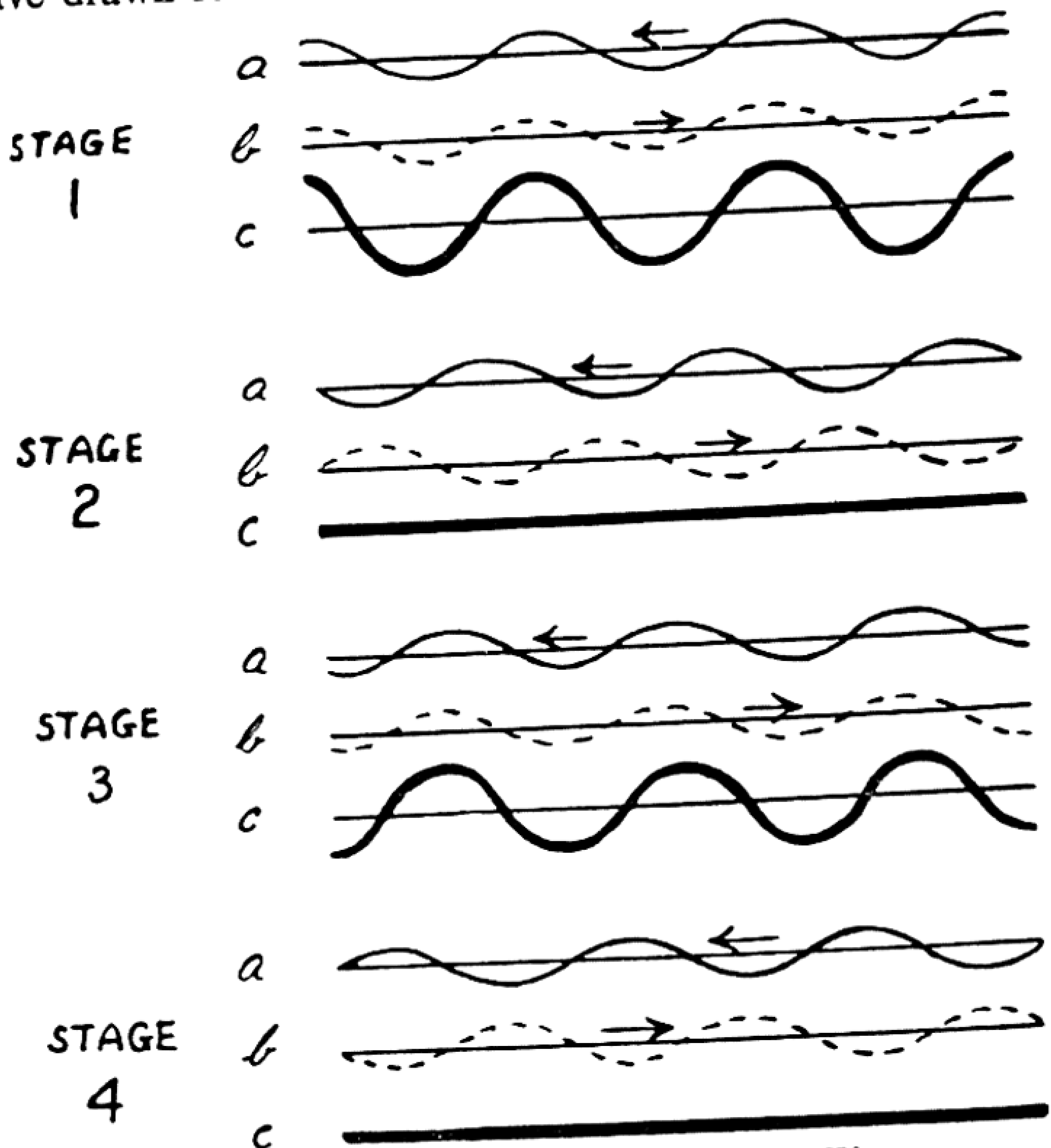


FIG. 40.—PROGRESSIVE AND STATIONARY WAVES.

waves, acting on a string of particles, first each alone, (*a*) and (*b*), and then superposed, (*c*).

Stationary waves form an important characteristic of bodies of limited length when set in vibration, as we shall see when we study sound.

Spreading of Waves

So far we have considered the waves to be propagated along a narrow channel, such as a cord. If a channel is narrow compared to the wave-length, the waves will pass along it with the wave-front as a crested line at right angles to the length of the channel and to the direction of propagation. Such an instance may be seen where the wind-waves on the sea or a lake pass up the entrance of a narrow tributary stream.

Such waves, where we find the whole of the motion in phase along a plane perpendicular to the direction of translation, are known as plane waves.

Where, however, the wave can spread laterally, we find the wave-front circular (if the spreading takes place over a surface, as in waves on the open surface of a pond) or spherical (if the spreading takes place equally in all directions, as in sound-waves from a source in the open air).

When energy is freely radiated in this manner, the energy arising in one second on unit area placed at a distance from the source—this quantity is often called the intensity—*falls off as the square of the distance from the source.* This Inverse Square Law, as it is called, arises from the fact that —in the absence of friction—the total energy crossing any sphere surrounding the source must be constant. As the area of surface of a sphere is proportional to the square of the radius, this means that the intensity for *unit* area of the sphere must vary inversely as the square of the radius of the sphere. As the energy is proportional to the square of the amplitude, the latter will decrease directly as the distance increases.

As the section of a sphere by a plane is a circle, the study of the behaviour of waves on a surface will give us a picture in two dimensions of spherical wave propagation such as we meet in the behaviour of sound-waves in the air, though the intensity will not fall off according to the same law. This idea is made use of in the study of the acoustics of buildings, to which we shall revert in a later section (p. 135).

Diffraction

When a sound-wave strikes a rigid wall with a hole in it, that portion of the wave which passes through the hole begins to spread on the far side of it. Actually it can be shown that its subsequent behaviour is as if a whole series of sources, conspiring in phase over this portion of the wave-front, started to send out spherical waves into the medium on the far side of the whole. Since the effects produced by these reinforce in front of the hole but interfere to each

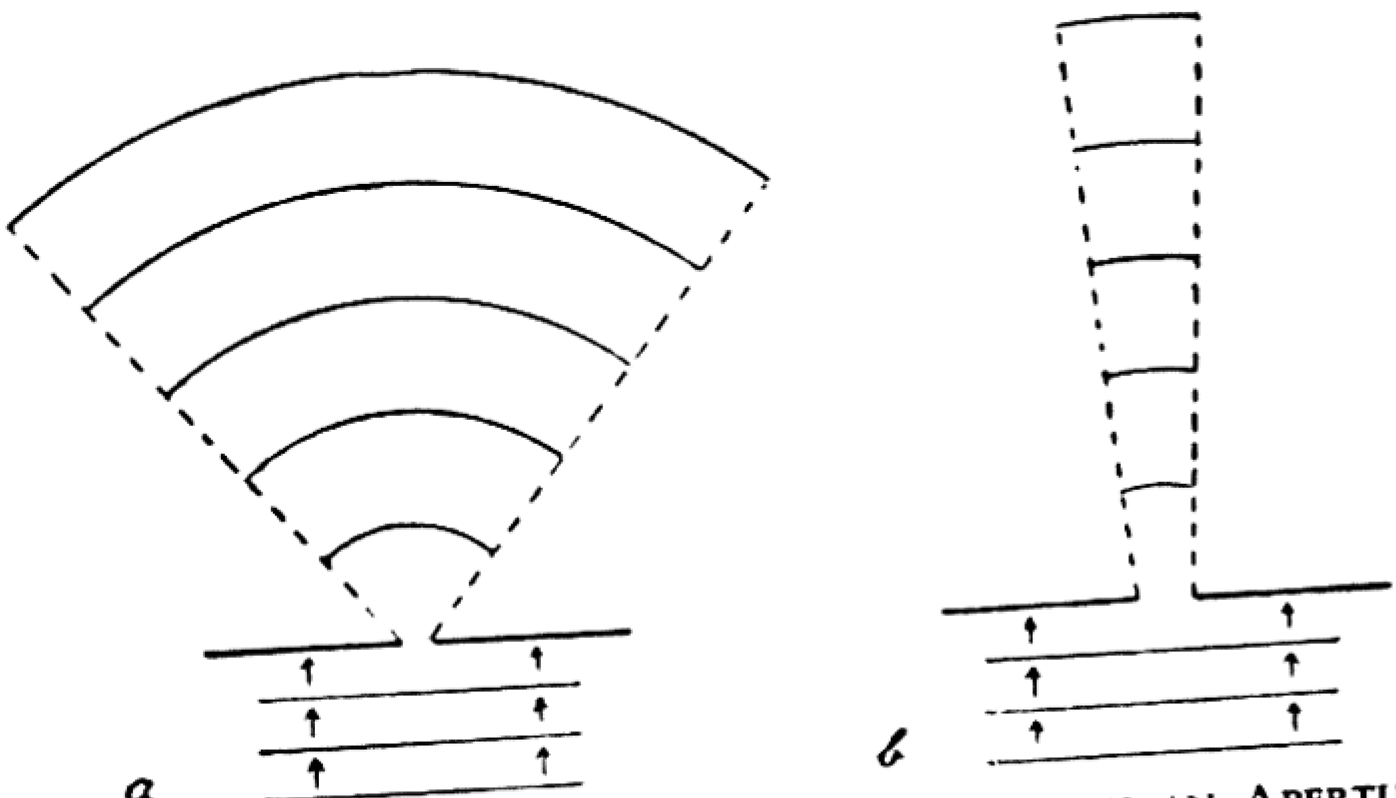


FIG. 41.—DIFFRACTION OF WAVES PASSING THROUGH AN APERTURE.

side, most of the energy is still carried forward in the direct line through the hole. It can be shown that the quantity which determines the amount of lateral spreading is the ratio of the diameter of the hole to the wave-length. If the hole is small and covers a few wave-lengths only, the hole acts almost as though it were a new source radiating energy spherically into the space beyond the hole, but if the hole covers a good number of wave-lengths, the energy goes straight forward as a *beam*.

Similar considerations apply to an obstacle. If this is large compared to the wave-length, little energy is found in the 'shadow', and vice versa. Spreading of this type caused by narrow holes and obstacles is known as diffraction.

Fig. 41 shows two examples of diffraction, where a series of plane waves strike a wall obstacle at right angles to the direction of propagation, with holes of diameter (*a*) small and (*b*) large compared with the wave-length.

It should be noted that for the purpose of exposition, these diagrams have been simplified in the following respects: (1) the waves reflected from the boundary are not shown, (2) the fact that along certain lines diverging from the edges of the hole interference occurs cancelling out the amplitude, has been suppressed. The latter rays, however, cover a small portion of the field and do not concern us in an elementary treatise.

Reflection

We have already discussed what happens when a periodic series of waves falls on a boundary between two media in a direction *at right angles* to the direction of propagation;

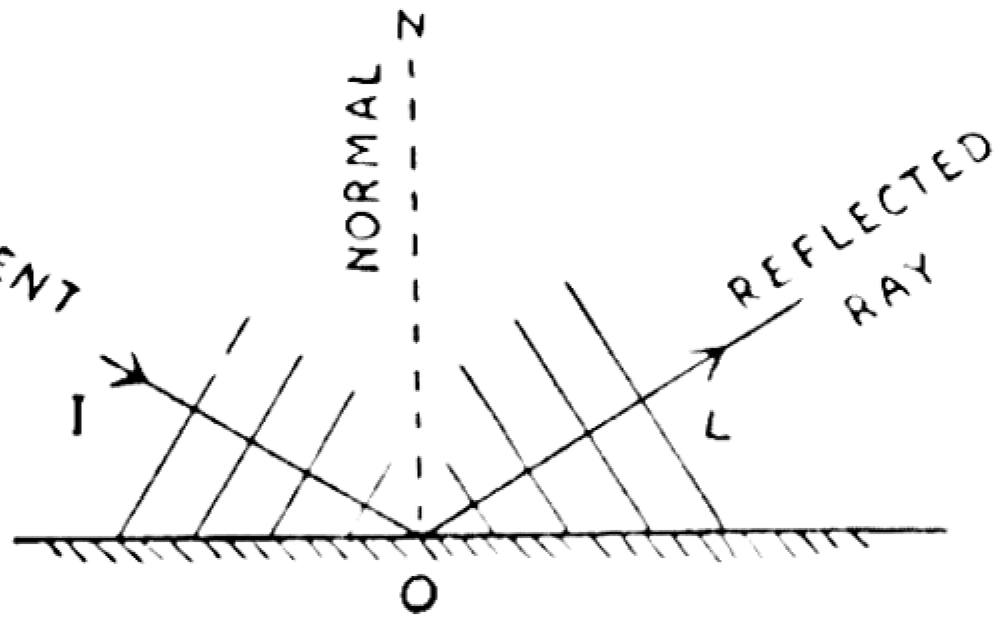


FIG. 42.—REFLECTION.

reflection causes interference and superposition of the waves to give rise to nodes and antinodes. If now the incident waves fall as a beam on a plane boundary *obliquely*, the wave-front is turned, as shown by the parallel lines in Fig. 42, and, except in the region where the two systems overlap and a certain amount of interference occurs, the beam sets off in a new direction, such that the directions of propagation of the incident and reflected waves make equal angles with the surface of separation. These directions are noted by drawing lines called rays perpendicular to the wave-front and

noting the angles which they make with the normal to the surface, a line drawn at right angles to the boundary where the incident and reflected rays meet it. The angle between the incident ray and the normal is called the angle of incidence (∠ION Fig. 42); that between reflected ray and

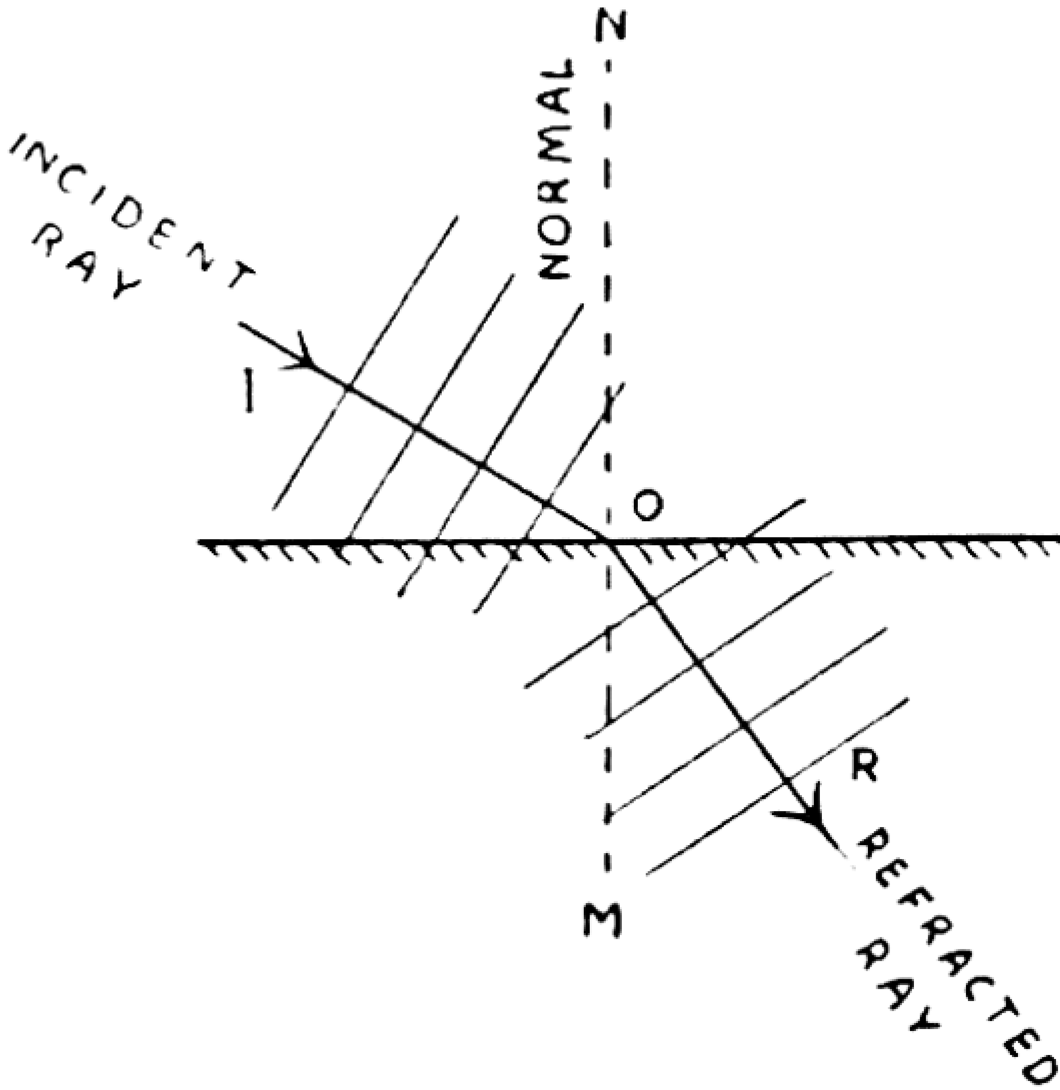


FIG. 43.—REFRACTION.

normal is the angle of reflection (∠LON) We may describe these observations by the two Laws of Reflection, viz.:

(1) the incident and reflected rays and the normal at the point of incidence lie in a plane;

(2) the angle of incidence is equal to the angle of reflection.

Refraction

If the medium behind the boundary is not completely impervious to the waves, a new set of waves called the refracted waves enters the medium along a ray from the point of incidence, making an angle with the normal called

the angle of refraction ($\angle$ROM, Fig. 43). This angle is not the same as the angle of incidence; its relation thereto depends on the ratio of the velocities with which the waves are propagated in the two media. Note that the frequency of the waves is unchanged, but that the velocity of propagation, and therefore the wave-length, is different in the two media. The velocity depends on the type of radiation concerned, whether sound or light or other form, and the properties of the medium, and will be discussed further in the appropriate places. For the present the reader should note the two Laws of Refraction, viz.:

(1) the incident and refracted rays and the normal at the point of incidence lie in a plane;

(2) $$\frac{\text{Sine of angle of incidence}}{\text{Sine of angle of refraction}} = \frac{\text{Velocity of propagation in first medium}}{\text{Velocity of propagation in second medium.}}$$

This ratio is known as the index of refraction.

Waves from a Source in Motion Relative to the Observer

If a source S' is emitting waves periodically at frequency f and these waves travel towards an observer O with a certain velocity V, while the source itself is moving towards the observer with a smaller velocity v, it will begin to catch up its own wave system. If the source had been stationary, then after one second from the time it started to emit there would be f waves ahead of the source covering a distance V (cf. Fig. 44 (*a*)), but due to its motion bringing it forward a distance v in the same time, these f waves are now compressed into a distance $V - v$ (cf. Fig. 44 (*b*)). The wave-length is reduced from $\frac{V}{f}$ to $\frac{V-v}{f}$, and the number which pass a stationary observer at O in one second is *increased* from f in the ratio of $\frac{V}{V-v}$.

Similarly, if the source moves away with velocity v, the f waves now fill a space $V + v$ after one second (cf. Fig. 44 (*c*)), the wave-length is increased from $\frac{V}{f}$ to $\frac{V + v}{f}$ and the number of waves passing O per second is *decreased* from f to $\frac{V}{V + v}$ times f. Note that in each case the number of

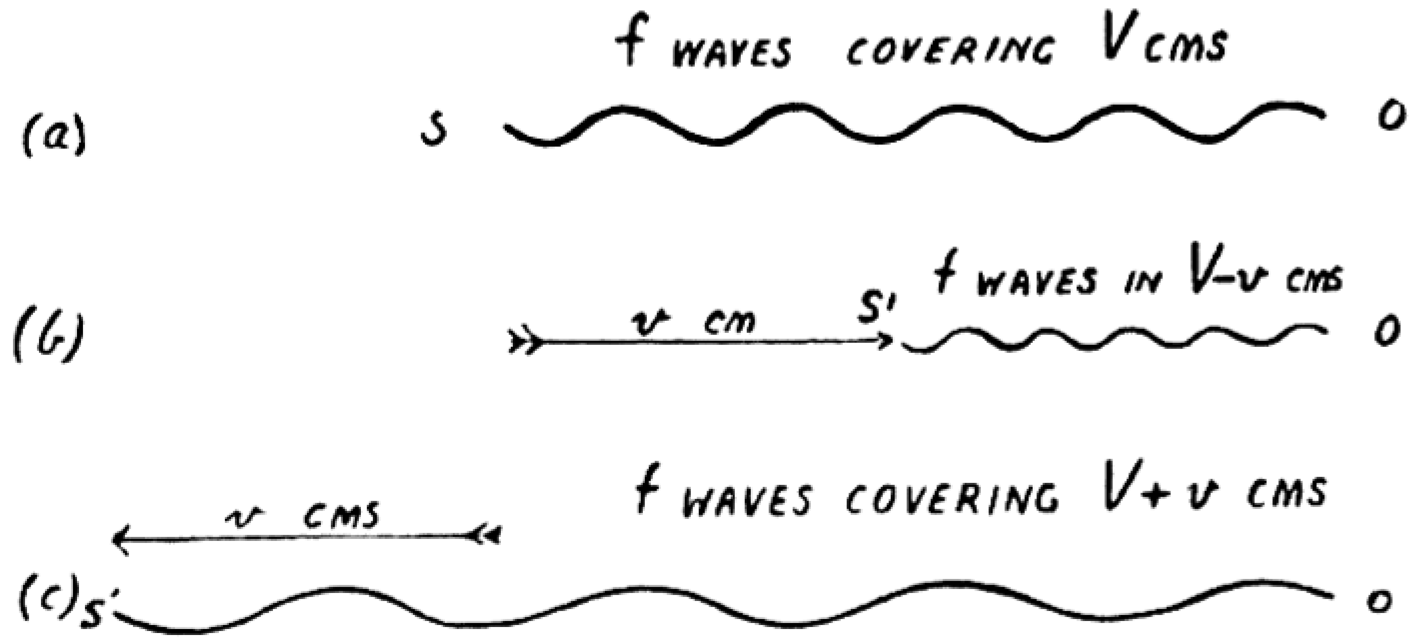


FIG. 44.—SOURCE OF SOUND IN MOTION.

waves passing the observer per second, or apparent frequency, is found by dividing the speed of propagation of the waves by the wave-length.

Similar considerations apply if the observer moves toward or away from a stationary emitting source, whenever, in fact, they are in relative motion.

This phenomenon is known as the Doppler Effect.

CHAPTER VIII

SOUND—GENERAL

The vibrations and waves involved in the production or propagation of sound are in the main transverse or longitudinal. The former type are usually found associated with the vibrations of solid, sound-producing bodies, the latter with fluids. The restoring forces are due to some form of elastic recovery. These may be imposed from without, as when a string is stretched by a force and plucked aside, or they may be internal forces of recovery, as when a bar of metal struck at one end emits sound.

Natural Vibrations

When a body is displaced and let go it is found that the frequency of the possible vibrations is determined by the possible systems of stationary waves which may be set up on it. It was explained in Chapter VII that these systems, which are called *modes* of vibration, are determined by the end conditions, i.e., whether fixed or free, since these in turn determine whether the end in question shall be a node or an antinode. That mode which has the greatest separation between a node and an adjacent antinode will have the greatest wave-length, and therefore the lowest frequency. It is known as the *fundamental* (mode). The incidence of other nodes and antinodes between the fundamental ones shortens the wave-length and raises the frequency. Such are called partial modes of vibration or, for short, *partials*. In simple vibrating systems, as we shall see, the frequencies of the partials form simple multiples of the fundamental; 1, 2, 3, 4, etc., though not all are possible with a given system. Ascending the scale of frequency, these will be known as fundamental or first partial, second partial, etc. The partial modes are also known as *overtones*. When the frequencies of the partial or overtones are related by whole multiples they are also called *harmonics*, being members of the harmonic series; 1, 2, 3, 4, . . . *n*.

Pitch and Loudness

Our sensation of the pitch of a note is governed by its frequency, in fact the words are often interchanged when hearing is concerned. In the same way, the loudness of a sound is determined by its physical intensity and therefore by the amplitude of vibration.

Quality or Timbre

We have written as though a system can vibrate either in a fundamental or one or other of its overtones, each one separately, but, in fact, unless special means of excitation are resorted to, it is likely to vibrate in a number of modes at the same time. The vibration will then be of the type whose form is indicated by Fig. 37 (*c*), (*d*), p. 96.

Physically the relative proportion of the different modes often depends on the strength of the exciting force acting on the body. Gentle excitation may result in the fundamental being sounded alone. As the exciting force is increased, more and more of the higher overtones are added and the fundamental amplitude decreases or vanishes altogether. The change in aural perception which accompanies these physical characteristics is known as a change in *quality* or *timbre*. The fundamental alone imparts a colourless but mellow quality, but as more harmonics come in, the tone gets brighter and fuller. If the admixture of higher harmonics is carried too far, the quality becomes metallic and harsh. *The relative phase of the components has no influence on quality.*

Concordance and Discordance

The importance of the harmonic series lies in this, that individual tones * from it sounded together or in succession give a pleasing relationship to the ear. Where overtones are inharmonic, i.e., their frequencies are not related by the ratio of small whole numbers, the resulting combination is unpleasant, or dissonant, to the ear. In music, not of the ultra-modern variety, it is therefore necessary to suppress

* It is generally agreed to call the sound resulting from S.H.M. a 'tone', that from several overtones together, a 'note'.

the inharmonic overtones of certain musical instruments, in order that they shall not clash with true harmonics.

Forced Vibrations

The natural vibrations of sounding bodies excited in the way we have described, by being displaced from equilibrium and let go, are, of course, damped vibrations. The initial amplitude decays at a rate depending on how much the system is subject to friction, because there is no maintaining force. If, however, one continues to act on the body with an arbitrary force, it may be constrained to vibrate in any frequency that this force imposes. Such vibrations maintained by an external source of energy are known as forced vibrations. The amplitude which may be imposed on the body by oscillating it by main force in this manner depends on a number of factors, but if the imposing force has its frequency gradually raised there will be certain frequencies passed through, at which the system is in sympathy with the forcing frequency applied to it, and the body will take up the amplitude with vigour. Such instances, wherein an excessive response on the part of the system is produced, are known as resonances, and they *occur when the frequency of forcing is nearly equal to one of the natural frequencies.* On either side of resonance as the forcing frequency is raised or lowered, the response rapidly falls off.

The phenomena of forced vibrations and resonance are well illustrated by a pair of simple pendulums adjustable in length and suspended side by side in a wooden framework (Fig. 45). Adjustment of the natural period of one of these may be effected by moving its bob along a short, threaded piece of metal. In order to vary the rapidity and strength with which the motion of one pendulum is imposed on the other, a noose is provided which can be slid up and down the suspension threads. When the noose is near the top the systems are loosely *coupled*, but as it is moved farther down the coupling becomes taut, and if one is maintained in to-and-fro motion it soon forces its oscillation on the other. If we use the left-hand pendulum (which has the

heavier bob) as impressed force and the lighter right-hand one as the patient on which the forcing is impressed, and first make them of unequal length, then on setting the left-hand one in oscillation at its natural period, it will begin to force the other to oscillate in time with it, in spite of the fact that the right-hand one has naturally a different time-period. This it will do the more readily if the coupling is tight. If the lengths of the two pendulums be adjusted to equality, we shall observe that the right-hand one exhibits a large response. This is resonance.

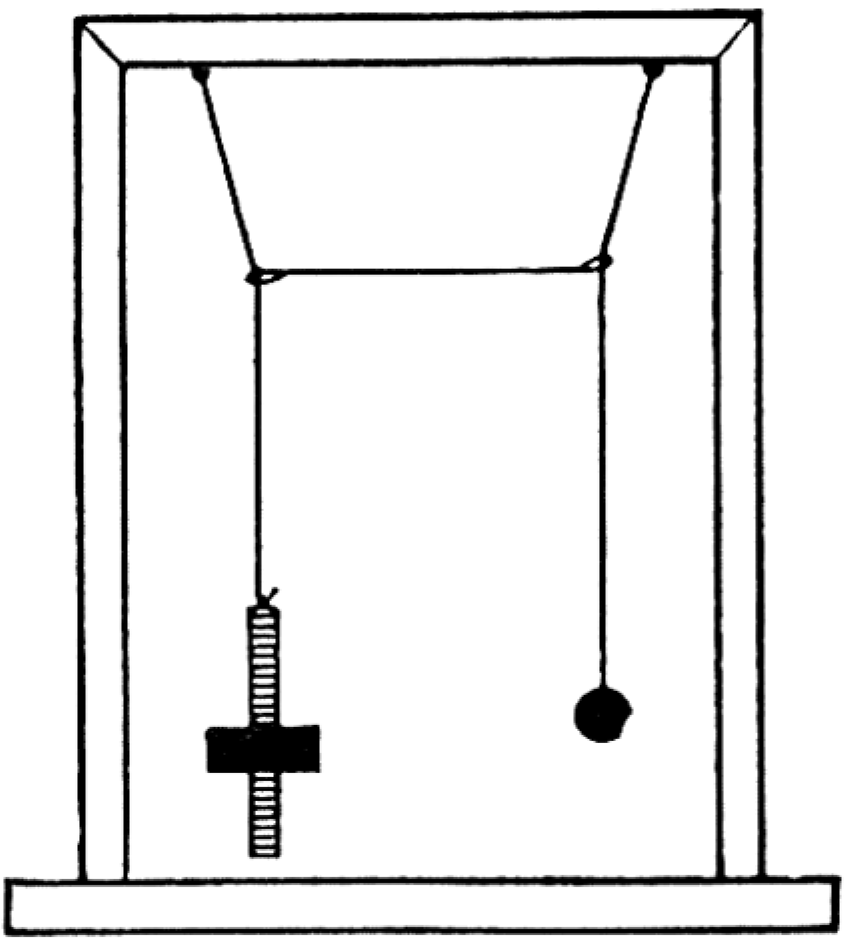


FIG. 45.—APPARATUS FOR SHOWING RESONANCE.

It will also be noted that, at resonance, the two systems do not move together, although they have the same frequency. The pendulum which is receiving the energy *lags at resonance* one quarter of a vibration behind the one which is emitting the energy—that is to say, that as the bob on the left-hand side passes through its lowest position, the right-hand one is just 'reversing' at the extremity of its swing. (This can best be seen by viewing the motion from one side, along the link joining the two pendulums.)

If the heavy pendulum on the left is replaced by one of

approximately equal mass to the one on the right and the two tuned by adjusting the lengths of the threads to equality, another feature of coupled systems may be studied—namely, the reaction of the forced system on the driver. If one be set in motion, it starts the other, but in doing so loses energy to the latter until it is *reduced to rest*, while the second has taken over all its energy. Then the second starts to react on the first again, till it has handed back this energy and reduced itself to rest. In this way they continue, each acting as driver and slave in turn. At any instant the one which is being forced can be distinguished, because it lags behind the driver. This feature of coupled systems was not so noticeable, though it existed when one bob was much heavier and therefore less prone to lose energy.

CHAPTER IX

SOUNDING BODIES

We have already remarked that bodies of finite extent can be set in one or other modes of natural vibration characterised by the position of nodes and antinodes distributed along them. In this class will be found most of those sources of sound in which one dimension is much longer than the others, as in stretched strings, rods or bars, columns of air in pipes. On membranes and plates which have only one dimension small the points forming nodes and antinodes are replaced by *lines*; still more complex systems with nodal *planes* may be set up when all three dimensions are of about the same size, as in the air enclosed in a room.

Stretched Strings

1. Transverse Vibrations.—We have already explained how the rate at which a body vibrates is governed by the *ratio of the elastic force* involved when it is displaced *to the inertia of the system*, in fact, it is found that the velocity with which sound-waves pass through a medium is given by the square root of this ratio. As a light string possesses no elasticity of its own, it must be stretched between two bridges by a force, which is supplied by the tension of a peg in a wooden hole (as in the violin), or by a weight hung on the end of the string, if it is supported so as to hang vertically. The latter method is preferred in physics, since the weight then measures the tension on the string. The inertia in this case is represented by the mass of unit length of the string. We may then write:

$$\begin{array}{c}\text{Velocity of Transverse}\\ \text{Waves on String}\end{array} = \sqrt{\frac{\text{Stretching Force}}{\text{Mass per Unit Length}}}.$$

(Note that if the denominator of this expression is expressed in grammes per centimetre, the numerator must be expressed in dynes, in order to deduce the velocity in centimetres per second.)

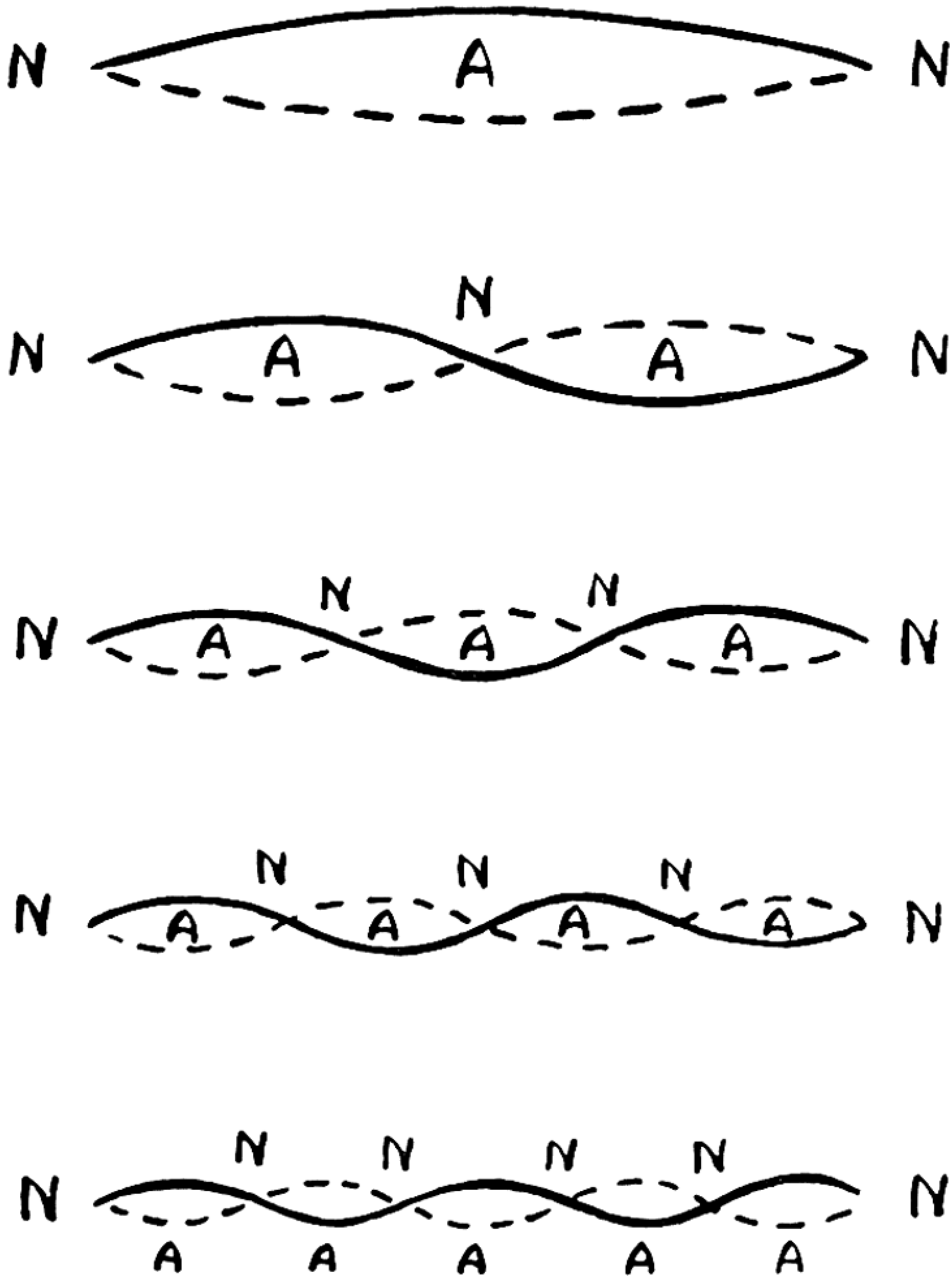


FIG. 46.—STATIONARY WAVES ON A STRETCHED STRING.

Since the ends of the string must be held taut in order to stretch it, they are not free to move laterally. The end conditions of a stretched string are accordingly: both ends fixed.

The lowest five of the possible modes of stationary vibration are shown in Fig. 46. N marks a node, A an antinode. If the length of the string be l, the wave-length corresponding to the fundamental is $2l$ and the frequency of this mode is $V/2l$, where V, the velocity, is given by the expression above. The next partial has an additional node in the centre; the wave-length is l and the frequency is $\frac{V}{l}$ or $\frac{2V}{2l}$. The third mode has two intermediate modes; the wave-length is $\frac{2}{3l}$ and the frequency is $\frac{3V}{2l}$. (For the method of deriving these lengths and frequencies, refer to Chapter VII.)

It thus appears that the possible modes of vibration of a stretched string in transverse vibration are (1) harmonic, (2) embrace the whole series of frequencies whose ratios to the fundamental are 1, 2, 3, 4, 5, etc., to infinity.

2. Longitudinal Vibrations.—It is possible to set a stretched string in longitudinal vibration by rubbing a resined cloth along it in the direction of its length. The velocity of these waves is much higher (cf. the section on rods, below), but the possible distribution of nodes and antinodes is the same as for transverse waves.

Melde's Experiment

In order to exhibit the modes of vibration of a stretched string and, incidentally, to illustrate the phenomena of resonance and coupled vibrations, we may use as exciting force a large tuning-fork or, better still, an electric motor with an eccentric boss on the axle which presses on the string and as the axle rotates gives it periodic impulses near one end, causing transverse waves to travel along it. The distant end of the string (which should be at least 6 feet long) passes over a loose pulley and has weights attached to its extremity to provide the tension (see Fig. 47). As the

motor is speeded up—it should be provided with a resistance for this purpose—resonance occurs whenever the revolutions per second coincide with one or other of the natural frequencies of the string. In the figure, resonance with the fifth harmonic of the string is pictured. When out of resonance no marked nodes or antinodes are produced and the amplitude is liable to vary rapidly. By measuring the distance between alternate nodes, the wave-length of the

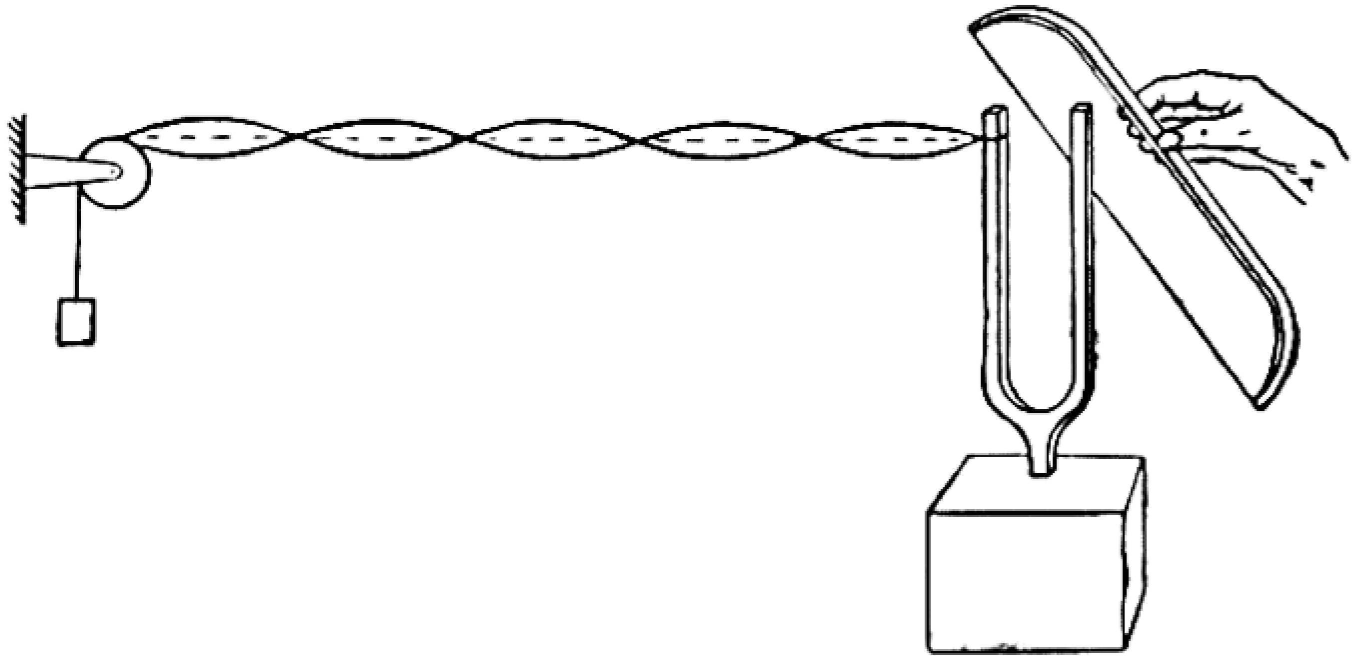


FIG. 47.—TUNING-FORK AND STRETCHED STRING.

note may be estimated, and so—if either the frequency of the fork be known or the rate of revolution of the motor be obtained from a counter on the axle—the velocity can be derived from the product of the frequency and the wave-length. If, further, a short length of the string be weighed to get the mass of one centimetre of it and the tension in dynes derived by multiplying the mass on the end of the string by the acceleration due to gravity, the formula given in the preceding section may be checked.

Applications of Stretched Strings

Strings in musical instruments are commonly stretched across a *sound-box* or *sound-board* with the object of reinforcing their rather feeble sound. Here again we find the principles of coupled systems applied to make the board or air cavity vibrate in sympathy with the string. In most instruments a number of tones must be produced, and since

these have all to be more or less equally reinforced by the same board or cavity, it is undesirable that these should have marked resonances in the range to be covered, or some tones will be exaggerated out of proportion to others.*

The ways in which the string may be set in transverse vibration are set out below:

A. *By Plucking.*—Part of the string is pulled aside by the finger or a hard substance. This type is found in the harp and guitar.

B. *By Striking.*—The string is hit at some point with a light hammer, which then bounces off the string, as in the pianoforte .

C. *By Bowing.*—The bow has a set of hairs which, with the application of resin to assist friction, is drawn continuously across the string in one direction. This displaces the string, gripped by the hairs, so far until the tension becomes too great to allow of further displacement, when the string, released from the bow, slides back until again caught up when it has lost all its momentum. Here we have an instance of a continuous (one-way) motion setting up an alternating one. It further differs from the two methods first mentioned in that the excitation (and therefore the amplitude of the vibration) is maintained until the bow is taken off the string, whereas in the harp and pianoforte the vibration is *damped*; the amplitude decays after the initial impulse.

D. *By Blowing.*—If a current of air is blown on to the string from one side, the disposition of the eddies which are produced in the lee of the string are such as to set it in vibration. Although such types of excitation are common in nature, they are not applied in musical instruments.

Stretched Membranes

Membranes (of paper, parchment, metal leaf, etc.) are commonly either circular or rectangular. They are usually

* The reader interested further in the musical applications of the principles discussed in this and succeeding similar sections should consult 'The Acoustics of Orchestral Instruments and of the Organ', by E. G. Richardson (E. Arnold & Co.).

stretched and pegged down over a framework, whose circumference thereby becomes a *nodal line*. Other lines, both perpendicular and parallel to the edge, are set up in the partial modes.

The fundamental of the circular membrane has a node round the circumference and an antinode at the centre. Partial nodes of higher frequency arise with (*a*) nodal circles intermediate between the centre and the circumference, (*b*) nodes along diameters, (*c*) superposition of both of these sets.

Applications of Stretched Membranes

The common way of exciting a membrane into damped vibration is by striking it with a padded hammer or with the knuckles, as in drums and tambourines. They are often stretched over resonant cavities of air (kettle drum). Stretched membranes are also found in loud speakers, in which a sound to be amplified is conveyed through a style attached to the central point. The style is agitated by electrical means. Here is another case in which overall response (without marked resonance) is aimed at.

In connection with the excitation of strings and membranes it should be pointed out that the *point* at which it takes place has an influence on the quality of the sound emitted, since it may favour certain overtones more than others. For instance, if we choose as our place of plucking, striking or bowing a string a point one-seventh of the length from one end, the seventh harmonic (with a half wave-length equal to one-seventh of the length of the string) requires a node or point of no motion at this point. This it is not likely to get if we give this point a large initial amplitude. Hence the seventh harmonic is found to be missing in the resultant sound when analysed. This case has been mentioned, because it actually occurs on the pianoforte. For reasons which we cannot go into here, the seventh harmonic is dissonant to the musical scale used on keyboard instruments, so the maker eliminates it by choosing as striking point for the hammers one-seventh of the length of each string from one end.

Transverse Vibrations of Bars

Applications.—A bar of wood or metal has an internal elasticity and requires no stretching force before it will vibrate. The velocity of transverse waves along it is a rather complicated function of this elasticity, its density and its thickness. It is usual to grip the bar at one end and deflect the other; on letting go, damped vibrations ensue.

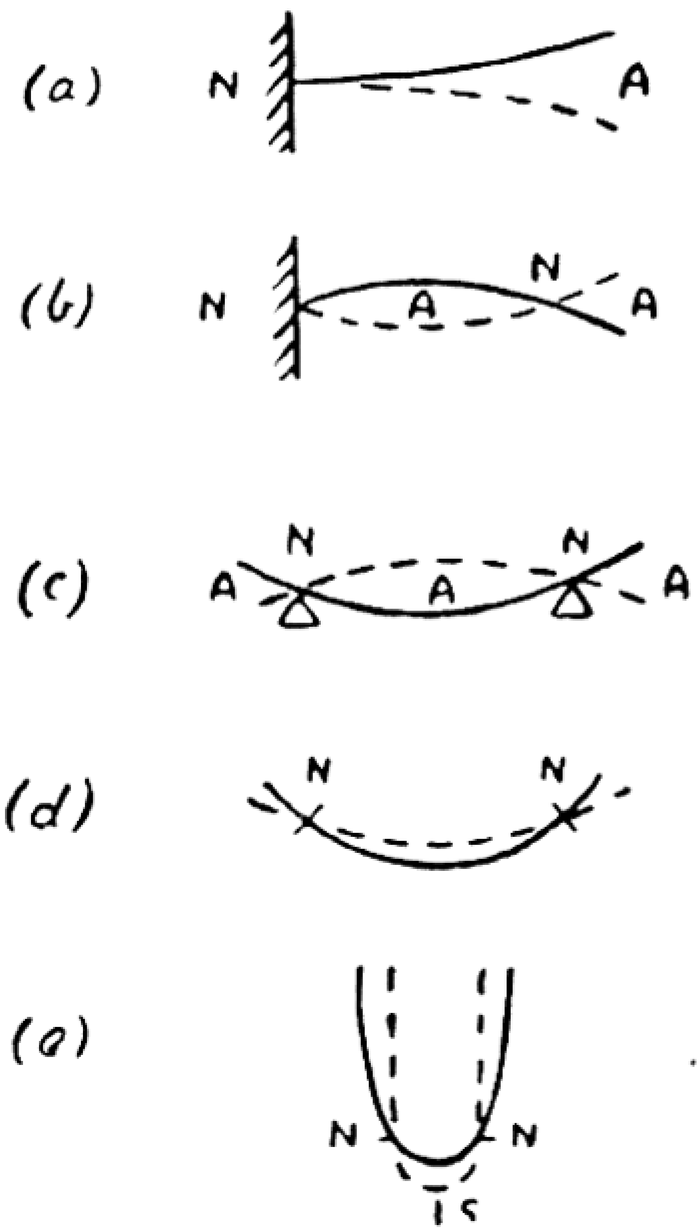


FIG. 48.—VIBRATING BAR AND TUNING-FORK.

By varying the length it is found that the frequency in transverse vibration is inversely proportional to the square of the length; but the overtones are not harmonic, bearing incommensurate ratios to the fundamental, although they are produced by subdivision into aliquot parts. Fig. 48 shows at (*a*) and (*b*) the fundamental and first overtone of a bar *fixed* at one end and *free* at the other. Thus the wave-

lengths or overtones of the bar are in inverse ratio of the series of numbers, 1, 2, 3, etc., as we go up the scale of the partial modes, but the frequencies do not increase in these proportions, because, in fact, the velocity of sound for each overtone in the bar is different.

Besides fixed and free ends, we may have a bar *supported* at ends or points. At such places, though the bar is not supposed to jump off its support, it is free to flex at this point. At (*c*) on Fig. 48 is shown a bar supported at two points one-quarter of its length from each end.

Bars clamped at one end are found in a number of musical instruments. In the harmonica they are mounted over resonance chambers and struck by hammers. In the harmonium they are mounted as reeds and set in vibration by the wind blowing over their tips.

Supported bars of wood are found in the xylophone, also mounted over resonant cavities, and struck by a hammer.

The tuning-fork may be regarded as originating from the supported bar, vibrating with two nodes. Suppose we gradually bend this up—cf. Fig. 48 (*d*)—then the nodes come closer together, until at (*e*), where the two sides are parallel, they are near the foot of the U. At this point the stem, *S*, may be inserted as indicated. If the fork is excited by plucking the ends together, the sound may be communicated to a table through the stem, which is nearly at an antinode. Large tuning-forks are often mounted, through the stem, to a tuned air cavity in a soundbox to enhance the sound at the expense of a more rapid damping.

The plate in vibration may be regarded as an extension of the bar in the direction of its breadth. For demonstration purposes, plates are usually held by a stem inserted at the centre. The frequencies of the overtones are, of course, inharmonic. These may be excited in turn by bowing on the edge at one point (which becomes an antinode) and touching at another (which becomes a node). The existence of nodal lines on the plate may be shown by strewing sand over the surface. The sand collects along the nodes and forms the Chladni figures, named after their discoverer.

Vibrating plates are used in the orchestra as cymbals.

Small thin plates of steel, clamped round a circular edge, are used to pick up sound-waves coming from the air in the microphone. They are then converted into electrical waves of the same frequency, amplified and passed to a loud-speaker which has either a similar plate or a large membrane of paper-like material.

Longitudinal Vibrations of Rods

The appropriate form of elasticity governing these motions is that which comes into force when the material is pulled or compressed along its length, and is known as *Young's Modulus* (of Elasticity). The inertia term is simply the density of the solid. So we write:

Velocity of Longitudinal Waves in Solids =

$$\sqrt{\frac{\text{Young's Modulus}}{\text{Density}}}.$$

For experimental purposes the rod is clamped at its mid-point, and the ends are free. The fundamental, accordingly, has a node at the centre and two terminal antinodes; the wave-length then is twice the length, as in the stretched string. The second partial has two additional nodes one-sixth of the length from each end, and the wave-length is now two-thirds of the length; the next partial has a wave-length two-fifths of the length, etc. Then the natural frequencies are in the ratios 1, 3, 5, 7, 9, etc. The overtones are harmonic, but those of even order are missing, as in the ' stopped pipe ' discussed in the next section.

The rod is excited into longitudinal vibration by rubbing it in the direction of its length with a resined cloth (for wood) or moist cloth (for glass and metal rods).

Columns of Air in Pipes

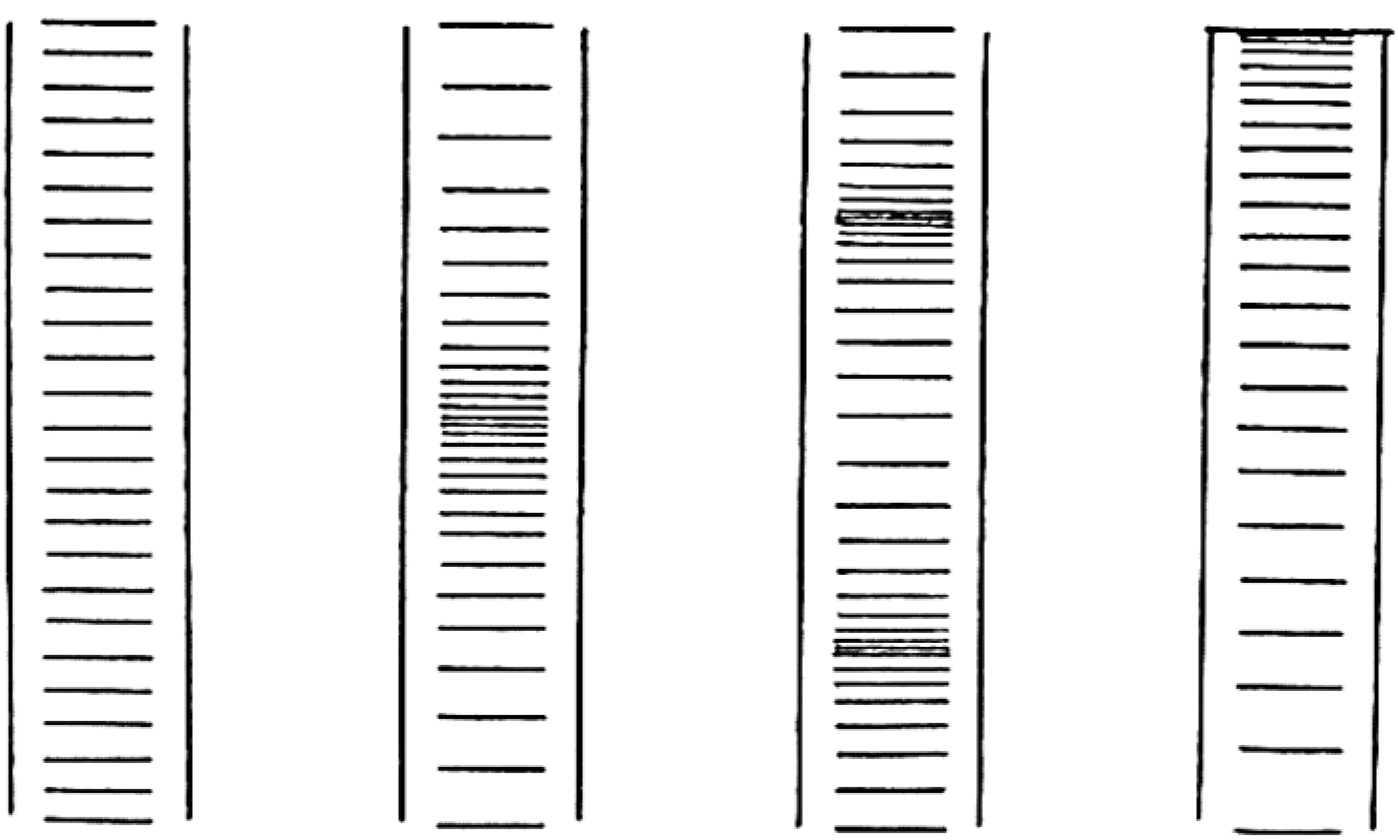
In this important class of sound-producers we are dealing with air (enclosed in pipes) set into *longitudinal* vibration. The velocity is still given by the square root of the quotient of elasticity by density. In the next chapter we shall discuss the value which has to be assigned to the density of the air in wave-motion. For the present we shall content ourselves with discussing the possible wave-lengths of such

columns. In order that the system can radiate the sound which it produces, one end, at least, must be open to the atmosphere. The other may be closed (fixed end) or open (free end).

The antinode at the free end of a reed may be justly located at the actual end of the material. At the open end of a pipe, however, the antinode is beyond the end of the material—that is to say, that waves travelling up the tube are reflected from a point beyond the end, and not from a point level with the top of the solid tube. This causes the effective length of the pipe to be longer than that of the solid material. *The amount by which the antinode overlaps the end is known as the* end-correction. For circular pipes without flanges the end-correction at an open end may be taken as three-fifths of the radius. No correction is necessary at a closed end where the node is terminated at the rigid wall or stopper.

Open Pipe

When the pipe is open at both ends its effective length is terminated by two antinodes. In the fundamental mode it will have a node at its mid-point. The air in the rest of



(a) (b) (c) (d)

FIG. 49.—MODES OF VIBRATION OF ORGAN PIPES.

the pipe moves to and from this node in concertina fashion, when the air oscillates. Fig. 49 shows at (*b*) the positions of layers of air whose undisturbed positions are shown in (*a*) —in the pipe at the phase of maximum compression. As with the rod clamped at its centre, the wave-length is then double the *effective length* of the pipe.

The next mode, shown at (*c*), has an antinode at the mid-point and two intermediate nodes. The wave-length is then the effective length (or one-half of double the effective length). The third partial mode has two antinodes at one-third and two-thirds of the effective length respectively. Its wave-length is one-third of double the effective length. And so forth.

Thus the relative natural frequencies of the column open at both ends cover the complete harmonic series, 1, 2, 3, 4, *etc.*

Stopped Pipe

When the pipe is open at one end and closed at the other it has an antinode at the open and a node at the closed end. The fundamental (Fig. 47 (*d*)) has a wave-length equal to four times the effective length.

In the next higher mode an intermediate node (at about one-third of the length from the open end) and antinode (at two-thirds of the length) appear. The wave-length of this mode is four-thirds of the effective length. The next one with two pairs of intermediate nodes and antinodes would have a wave-length four-fifths of the effective length.

Thus the relative natural frequencies of the column closed at one end cover the odd members only of the harmonic series, 1, 3, 5, 7, *etc. Further, the fundamental frequency is approximately one-half of that of a doubly-open pipe of the same length.*

The reader should appreciate the significance of the word 'approximately' in the above statement. Because the open pipe embraces two end corrections, while the closed pipe has only one, their effective lengths are not precisely equal when the length of tube involved in each is the same.

These facts are summarised in the following table, where l = effective length.

	Open Pipe				Stopped Pipe			
Mode	1	2	3	4, etc.	1	2	3	4, etc.
Wave-length	$2l$	l	$2l/3$	$l/2$	$4l$	$4l/3$	$4l/5$	$4l/7$
Frequency, relative to fundamental of stopped pipe	2	4	6	8, etc.	1	3	5	7, etc.

Two simple experiments illustrate these relations:

1. Resonance Tube.—Some means of varying the length of a pipe is needed. For the open pipe two brass tubes about 18 inches long, which telescope into one another with a tight fit, may be used; for the closed pipe a single tube 2 or 3 feet long, dipping into an equally tall cylinder of water. A tuning-fork of 'middle *A* or *C*' completes the equipment. Starting with the pair of tubes telescoped as far as they will go, or with the single tube mostly immersed in the water, and gradually increasing the effective length, a length will be reached (about 2 feet for the open and 1 foot for the closed pipe) at which, on striking the fork and holding it over the open end, maximum re-inforcement of the sound occurs, indicating that the pipe is in resonance. On lengthening the column a second point of resonance will be found, this time with the first overtone (second partial mode in the above table) at a length approximately double the first in the open pipe, and treble the first in the closed pipe. In either case *the increase in length* is that due to an additional segment in the stationary waves set up by the note due to the fork, and *is thus a half wave-length for the sound.* If the frequency of the fork is known, the velocity of sound in the column of air can be calculated.

2. Kundt's Tube.—In this experiment a rod in longitudinal vibration is set in resonance with a column of air also in longitudinal vibration. The rod (preferably of wood or glass) is clamped firmly to the bench at its centre (Fig. 50), and has a cardboard disc stuck on the end which fits loosely into a glass tube about 1½ inches wide, with an adjustable stopper. In order to show when the column of air in the tube is in resonance, lycopodium (a light powder

formed of the spores of a fungus) or cork dust is strewed along the bottom. The rod is excited by rubbing it near the free end with a cloth (coated with resin for a wooden rod; moistened for a glass rod), and the stopper moved near the far end of the tube, until resonance is indicated by the falling of the powder into regular patterns. In fact, when the column of air is excited into one of its partial modes of vibration, the powder will be vigorously disturbed at the antinodes (often to fall in regularly spaced bands

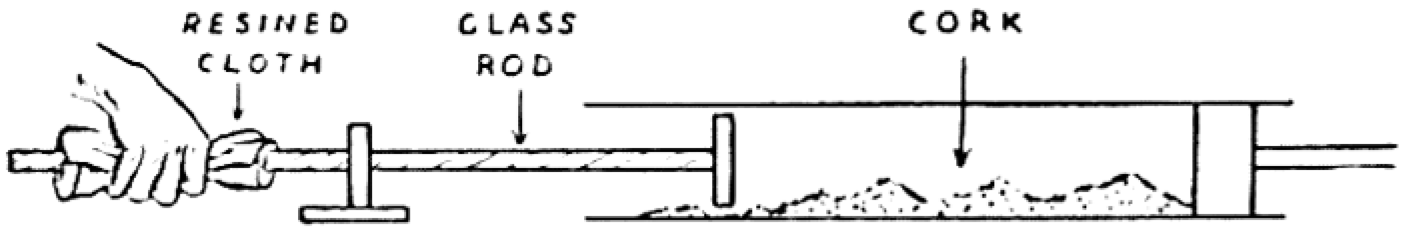


FIG. 50.—KUNDT'S TUBE.

across the bottom of the tube), but undisturbed at the nodes, where the amplitude of the air particles is zero. By measuring the distances between successive nodes the half wave-length in air may be found. The half wave-length in the rod at its fundamental (which is the only mode likely to be set up in this method of excitation) is given by the distance between the antinodes at its ends—that is, the whole length of the rod. Since it must be the same note in each, the ratio of these two lengths is the ratio of the respective velocities of sound in wood (or glass) and air. Since the latter is by far the smaller, the air column will be vibrating in one of its higher partial modes.

Applications of Air Columns

Columns of air in resonance at one or other of their natural frequencies are found in many musical instruments. Two methods of excitation are commonly employed.

1. Edge Tone.—An edge tone is produced whenever a thin stream of air impinges on a sharp edge. In the *diapason* class of organ-pipes air under pressure is blown out of a narrow slit to impinge on the sharply bevelled edge of one end of the pipe which encloses the air column. In the coupled system—edge tone plus column of air—the latter

is the predominant partner, so that within certain limits of wind pressure the fundamental is elicited. In the orchestral flute the player produces the thin band of air from his lips and sets it to strike against one edge of a hole cut near one end of the tube.

2. Reed.—The reed usually lies across an orifice, which it almost covers. Wind coming out of the hole deflects it outward, from which position it then relaxes and vibrates in S.H.M. The reed on its supply pipe is mounted at one end of the column of air which it excites.

Each pipe on an organ is intended to sound at one frequency, usually the fundamental. In the orchestral wind instruments a series of notes must be produced in turn from the same pipe. The variation in frequency is produced in two ways:

1. The column is made to give out an overtone. This is usually done by increasing the wind pressure.

2. The effective length of the column is changed by opening holes in the side so that the antinode is shifted from the open end to some intermediate point along the tube, thus raising the frequency.

Secondary Sound Sources

In the applications of sounding bodies to which we have up to the present referred, the apparatus has stationary waves set up in it and it vibrates in one of its natural frequencies. Often it is necessary to use such a system to amplify sound-waves which are fed into it. In such a case it is undesirable that it should have prominent natural frequencies, and ideally it should go to the other extreme and give equal response to all frequencies. Such a system cannot be attained in practice, but the way in which it is approximated to can be illustrated by reference to the gramophone.

This consists of three principal parts:

1. The record on which has been traced a groove so cut that when a stylus passes through it, it is given a to-and-fro or up-and-down motion corresponding to the wave-form of the music which is to be reproduced.

2. The sound-box in which the motion of the stylus is,

either directly or through the intermediary of electrical energy, conveyed to a diaphragm or thin metal membrane.

3. The horn in which the vibrations of the membrane are converted into aerial copies.

In this complex coupled system the fundamental modes of the stylus and membrane are placed as high as possible, so that they are not often excited. The fundamental of the horn should be low, which means a long air column or a large flaccid membrane, if this is used in place of a horn. Moreover, the horn has a large 'flare'. Instead of being cylindrical or slightly conical, as in orchestral instruments, it widens out so rapidly that the amount of sound reflection back into the pipe at the open end is not enough to set up the stationary waves which are a necessary accessory to resonant vibration.

The recording of the sound-waves is done with similar apparatus in a reverse process.

Mechanical reproduction of sound is now largely superseded. Electrical amplification is used and re-transformed into mechanical vibration at the throat of the loud-speaker. This may be done in several ways.

1. The sound-track is inscribed by a cutting stylus on a disc. The *pick-up* stylus moves in a magnetic field and the current induced in a coil is conveyed after amplification to a similar transformer on the loud-speaker diaphragm.

2. The sound-track is inscribed on a photographic film by varying the amount of light falling on it in synchronism with the sound-waves. In reproduction the film-record interrupts a beam of light falling on a photo-electric cell, and the current from the latter is impressed on the loud-speaker.

3. Electrical vibrations copying a series of notes are excited by electrical oscillators and superposed on the loud-speaker. The 'notes' are arranged like the keyboard of a pianoforte or organ. There is an oscillator to each note, but the quality can be varied as well as the fundamental pitch. Such systems, though they use the technique of electro-mechanical reproduction, are really primary sources of sound, and under the name of *electronic organs* are played as musical instruments.

CHAPTER X

TRANSMISSION OF SOUND THROUGH THE ATMOSPHERE

Methods of Measurement

To anyone who observes the onset of a distant factory or ship siren it is apparent that sound takes an appreciably longer time to travel than light. The velocity of sound in air is, in fact, negligible in comparison with that of light. This fact has been made use of for many years to measure the former quantity. The source is usually the exploding of a charge of powder or the firing of a gun of large calibre. Observers stationed on an eminence note on a chronometer the time which elapses between the receipt of the light and the sound signal. Allowance is made for the effect of wind by making observations in two diametrically opposed directions. In recent years the light has been replaced by a radio signal which travels with the speed of light and the sound has been picked up on sensitive recording microphones, the time interval being measured mechanically, but the principle remains the same.

The speed of sound in air can also be deduced from laboratory experiments in which the wave-length in air that corresponds to a source of known frequency is measured. Thus, the experiments with the resonance tube and Kundt's tube (p. 127) serve, if the frequency of the tuning-fork or of the sounding-rod be known, to give a value of the velocity of sound in the air in the tube. If the tube be narrow, this will be less than that in the open air, but with a source of frequency near that of 'middle C' and a tube at least 2 inches wide, the difference will be negligible.

Theoretical Value for Velocity in Air

Since sound is propagated as a *longitudinal* wave in air, we shall use the expression already given:

$$\text{Velocity of sound} = \sqrt{\frac{\text{Elasticity}}{\text{Density}}}.$$

It then becomes a question of what value to take for the *elasticity*. The latter is defined as the fractional decrease of volume for unit increase of pressure. Accordingly, we must assume some relationship between the volume and pressure of a gas, in order to derive an expression for the elasticity. Now, where a gas is compressed, some of the work done appears as heat. If, then, a local compression occurs, this part of the air is warmed; nearby, in the neighbourhood of a rarefaction, the air is cooled. If the compressions and rarefactions succeed each other very slowly, there is time for these heat changes to be levelled out by heat being conducted from the hot to the cold places, so that the variations of pressure and volume remain isothermal (= constant temperature). Under these circumstances we know that Boyle's law applies (see p. 39). The decrease of volume divided by the original volume is then proportional to the increase of pressure divided by the original or prevailing pressure. The fractional change of volume produced by a given increase of pressure is equal to the prevailing pressure. Thus *the isothermal elasticity is equal to the pressure* prevailing.

If, however, the alternations of compression and rarefaction occur so rapidly that the air is not able to conduct the local heatings and coolings from the places at which they are produced, the law which we must invoke in place of Boyle's law is that proper to *adiabatic* changes of volume—namely, that in which the heat produced in a stratum of the gas stays in it and cannot get out before the next volume change. The factor by which the isothermal elasticity must be increased to give the adiabatic elasticity proper to this case is equal to the ratio of the specific heats at constant pressure and at constant volume. This factor is 1·4 in the case of air.

If, then, we insert this value of the elasticity into the expression which heads this paragraph, 1·4 times the value of the normal atmospheric pressure (in dynes per square centimetre) and of the density of air (in grammes per cubic centimetre), we get as *theoretical value for the velocity of sound in air at N.T.P.*—330 *metres per second.*

Now, this agrees closely with the experimental value,

whereas that which we derive by using the isothermal elasticity is too low (280 metres per second). We must therefore conclude that the vibrations of sound are so rapid that down to the lowest frequency which can give rise to the sensation of sound, the changes in volume are adiabatic. This applies to all gases remote from their condensation conditions.

Meteorological Influences on the Velocity of Sound in Air

A. *Pressure* has no influence, since both the elasticity and density are equally affected by changes of pressure.

B. *Temperature* rise causes a rise of velocity, since by Charles' Law the density varies inversely as the absolute temperature (p. 42). Thus the speed of sound in a gas increases as the square root of the absolute temperature.

C. *Moisture in the Air and Winds* also have an influence on the velocity, the former by decreasing the density, the latter by adding or subtracting a component of its velocity in the direction in which the sound is heard.

Sound from a Moving Source

As long as the source of sound and the hearer are not in relative motion, the wave-length is unchanged. Thus if the source and receiver are still and a wind blows from one to another, although the speed at which the sound-waves travel towards the observer is not the same as in still air, yet he cannot appreciate any change of pitch (frequency), for the waves hit his ear as fast as they are given out by the receiver. If, however, he moves to or from the source, or the source moves towards or away from him, his ear picks up compressions at a faster or slower rate, respectively, than they are given out by the source.

When the two are in relative motion towards each other, the sound picked up *appears* to have a higher frequency than when both are still. When moving away from each other, the apparent frequency is lower. This is an application of the Doppler effect discussed on p. 109, and explains the sudden fall in pitch heard when one is standing beside a railway track as a locomotive passes by, blowing its whistle.

Refraction and Diffraction of Sound in the Atmosphere

When the atmosphere, instead of being of uniform meteorological condition over a large area, is stratified or ' patchy ' in respect of temperature, wind, or fog, sound-waves are no longer propagated in a forthright direction, but are bent to one side. The simplest case to consider is that in which there is, say, a layer of warm air overlying one of cold air near the ground. Then, as sound-rays pass up from a source near the ground, they are refracted on reaching the warm zone in a direction away from the normal —that is, become more inclined from the vertical, since they are entering a region where the velocity of sound is higher (cf. p. 108) than it was lower down.

Conversely, if a cold stratum overlies a warm one, sound going up is bent towards the normal—that is, becomes more nearly vertical.

These considerations are of importance when sound is propagated from a powerful source—for example, an explosion—to great distances. Under normal conditions the temperature falls slowly in the first few miles going up, so that a sound-ray, starting initially at, let us say, 45° to the horizontal, is *gradually* bent to a steeper inclination. Above 10 miles, however, the temperature fall comes to a halt and begins slowly to rise. This inclines the rays towards the horizontal again until they suffer *total internal reflection* (cf. p. 156) at some stratum when they come down eventually to the ground. Rays which have gone out horizontally, grazing the earth's surface, however, are usually reduced to inaudibility in about 50 miles, whereas the returning refracted ray may come down with sufficient intensity to be heard at 100 miles or more from the source. This explains why the firing of heavy guns may sometimes be heard at 25 miles and 100 miles from the source but not in the intervening zone of silence, as it is called.

Refraction of sound-waves may also occur when they pass through strata in which the moisture content suddenly or progressively changes or where the wind velocity changes. In each case the refraction is caused by the rays passing into

a region where the local velocity of sound differs from that in which it formerly passed. When a gradient of wind is in question it is the change in the algebraic sum of sound and wind speed which is the cause of the refraction.

Diffraction ensues wherever there are obstacles—the so-called acoustic clouds—in the atmosphere whose size is comparable with the wave-length of the sound concerned. These may be solid bodies, banks of moisture or of hot air, or even eddies in which there is a discontinuity in property sufficient to cause bending of the rays.

All the laws with regard to reflection, refraction and scattering of radiation which are stated in this book in reference to heat and light propagation apply with equal force to sound, having regard to the *much greater wave-length associated with sound.* Since most terrestrial obstacles embrace only a small number of sound wave-lengths, bending of sound round corners is much more common. In light the opposite is true, hence the dictum that 'light travels in straight lines'. To act as a mirror to concentrate sound we must use one which is several feet across, unless we are concerned with a source of very high pitch, otherwise our efforts will be offset by the diffraction produced by the obstacle, and no focusing will occur. Similarly, in order to break up a mirror surface, for example, a domed vault, large excrescences or indentations several feet wide must be made in the smooth, curved surface, if focusing of sound at some point below the dome is to be avoided.

Acoustics of Interiors

The points made in the last paragraph bear an important relationship to the way in which sound is distributed inside a room. In order that every auditor in a large hall can hear clearly, it is important that the sound should be evenly distributed in the air-space. Large, unbroken, curved surfaces tend to concentrate the sound at certain points to the detriment of others. Further, although a listener at such a focus gets more than his fair share of sound, in virtue of the fact that it is reflected sound, it may arrive appreciably later than the direct sound from the same source, and so

produce a disturbing amount of overlap or echo. On both counts such surfaces are undesirable.

Another factor which intervenes in the possibility of correctly appreciating a speech or a piece of music given in an auditorium is the reverberation in the building. In the open air the sound from each note or syllable passes each listener once only (unless reflected by neighbouring walls or cliffs), but in an interior it is continually ricocheting between the walls, floor and ceiling until damped out. A certain amount of such reverberation is desirable, as it adds to the loudness which the listener picks up—additional energy which he would not get out-of-doors—but it does harm by prolonging the time during which this note is still audible in the room. Excessive reverberation therefore makes a speech difficult to hear and gives a 'smudgy' rendering to any music played in a hall with such characteristics.

Correct reverberation is attained by using special absorbent materials on the wall which produce such a loss of energy at each reflection that the time for which a sound of normal intensity is still audible is proportioned to the size of the room.*

* The reader desiring further information on this topic is referred to 'Acoustics of Buildings', by E. G. Richardson (E. Arnold & Co.).

LIGHT SECTION

CHAPTER XI

REFLECTION OF LIGHT

The Nature of Light

At the conclusion of the section on heat we were discussing the nature of radiant heat, and we pointed out that as a body has its temperature raised, so it passes from its normal non-luminous state to red heat. The next stage as the temperature is raised further is that the hot body acquires an orange tint, then yellow, and finally becomes white-hot. These changes may be seen, for example, in a poker which is thrust into a fire of hot coal or, in reversed order, after the poker is made white-hot in a furnace and then brought out to cool. During all the stages beyond that at which it glows with a dull-red tint, the body is self-luminous and is actually emitting both heat and light. From this we conclude that heat and light are both essentially of the same nature and can travel through a vacuum—a fact we know because both heat and light reach us from the sun. Apart from bodies like the sun at one extreme and the glow-worm at the other and the products of burning fuel which are in themselves luminous, we observe non-luminous bodies like trees and houses and the moon from the light which they reflect into our eyes. Light is a disturbance of the aether, occupying all space, which, though it does not involve the motion of material particles, can be explained in many of its aspects as though it were a wave-motion of very high frequency but small wave-length, the disturbance being transverse to the direction in which the light is travelling. Further than this rather unsatisfactory description of the nature of light we cannot go in an elementary book, but fortunately most of the observations on light which we wish to establish can be satisfactorily accounted for on a simpler basis: '*Light travels in straight lines*'.

It is a matter of common observation that a small source of light such as a flash-lamp bulb casts sharp shadows of objects on which its light shines. If such a pea-lamp is placed on the table at one end and a book upright at the other, the shadow is the *projection* of the book on the wall—that is to say, that imaginary lines drawn from the bulb to the edges of the book would strike the wall at the confines of the shadow, as though the little source of light were sending out straight lines of rays in all directions, some of which are stopped by the book wherever they fell upon it. This idea of a light-ray as something shot out of the source is also exemplified by the toy apparatus known as the 'pinhole camera'. This may be fashioned out of a card-board box, one end of which is pricked in the centre with a pin, while the opposite end is removed and replaced by a piece of frosted glass to act as screen. If this be pointed at a distant tree, an image or picture of the tree will be formed upside down on the glass screen. This is because every leaf of the tree sends a ray of light—reflected sunlight —through the pinhole to fall on the screen at a definite point. The higher up the tree that the leaf is, the lower down on the screen will the ray strike—hence the inverted image. (The observer's head and the back of the camera should be covered with a black cloth when observing the image.) If the pinhole be widened, the image becomes blurred. This is because a bundle of rays from each leaf or point on the stem can now get through the hole to fall on the screen. The point-to-point relationship which was a requisite for the production of a sharp image is then destroyed. To each point of the tree there now corresponds a patch on the screen. The overlapping of these patches causes the blurring. Finally, if the hole is very wide the overlapping is so great that nothing but a light indistinct spot is formed on the screen.

A similar blurring of a *shadow* occurs whenever the source of light, instead of being concentrated at a point, has considerable width compared to that of the obstacle which is casting the shadow. It will be noticed, for instance, that the shadows cast by a hot coal fire or furnace of objects in

its vicinity have blurred edges. This is because each part of the fire casts its own shadow of the obstacle (according to the straight-ray rule), and these shadows are therefore different for different parts of the source, and overlap, giving a dark central portion to the shadow and a greyish shading off at the edges.

We conclude, then, that ' light travels in straight lines ', and we shall see that this statement is adequate to describe most of the phenomena with which we are concerned. The limitation to which the statement is subject will be discussed in the final chapter.

Optical Apparatus

Our principal concern will be to demonstrate the laws of reflection and refraction (see p. 107) in relation to mirrors, prisms and lenses, and to discuss their application. In this connection, the source of light (whether self-luminous

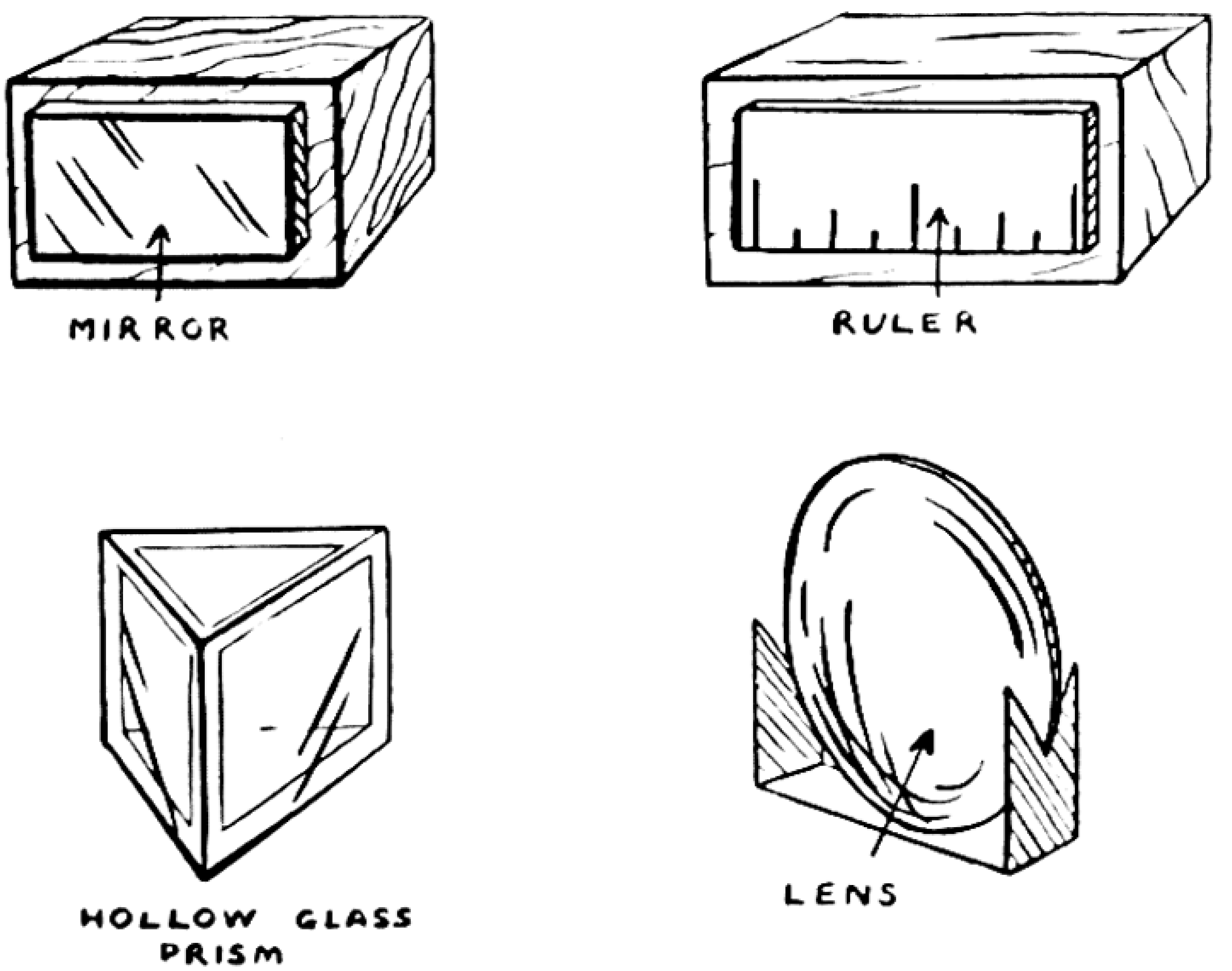


FIG. 51.—SIMPLE OPTICAL APPARATUS.

or sending out again any daylight which falls on it) is known as the *object*. If rays sent out from the object, after reflection or refraction, concur again to form a picture of the original object, this picture is known as an *image*; compare the operation of the pin-hole camera just described.

The student should equip himself with a small pea-lamp or flash-lamp bulb to act as luminous object and a white card on which to receive the image. For non-luminous sources, and also to trace rays, a few large pins are required.

Mirrors may be made by silvering pieces of glass—flat pieces for plane mirrors, watch-glasses for curved mirrors—or by bending pieces of shiny tinplate into suitable shapes. The plane pieces should be tacked on to small blocks of wood so that they can stand on the table at the height of the pin-heads. The curved pieces of tinplate will stand up on their edges. The silvered watch-glasses should be made to stand up on their edges by gripping them in pieces of bent-up tin (see Fig. 51).

Lenses—one convex on both sides and one concave on both sides—can be obtained quite cheaply, and mounted in similar tin-holders. If a glass prism is not purchased, it is possible to make a hollow prism by cutting a thin glass triangular base and three vertical sides, stuck together at their edges by narrow strips of surgical tape. This prism can then be filled with water.

PLANE MIRRORS

Tracing Incident and Reflected Rays

Place the mirror with its face vertical on a piece of white paper on the table. About 2 inches away, and a little to one side, place the pin, *O*, to act as object. Looking into the mirror, the image is seen. Since this is 'through the looking-glass', it is not real in the sense that it can be received on a screen (like the image in the pinhole camera), but is merely the point from which rays appear to come from the object after reflection into the eye. If we can trace a number of such rays, we can find the point from which they are diverging, and so find the position of this *virtual image*.

To do this place another pin at P_1, nearer the mirror (Fig. 52). The line OP_1 now defines a ray of light coming from the mirror. By joining OP_1 with a straight line and producing it until it strikes the mirror, we find that this ray is incident on the mirror at the point Q_1. Now look into the mirror with the eye until you see the reflections of the pins O, P_1 in line with the point Q_1. You must then be

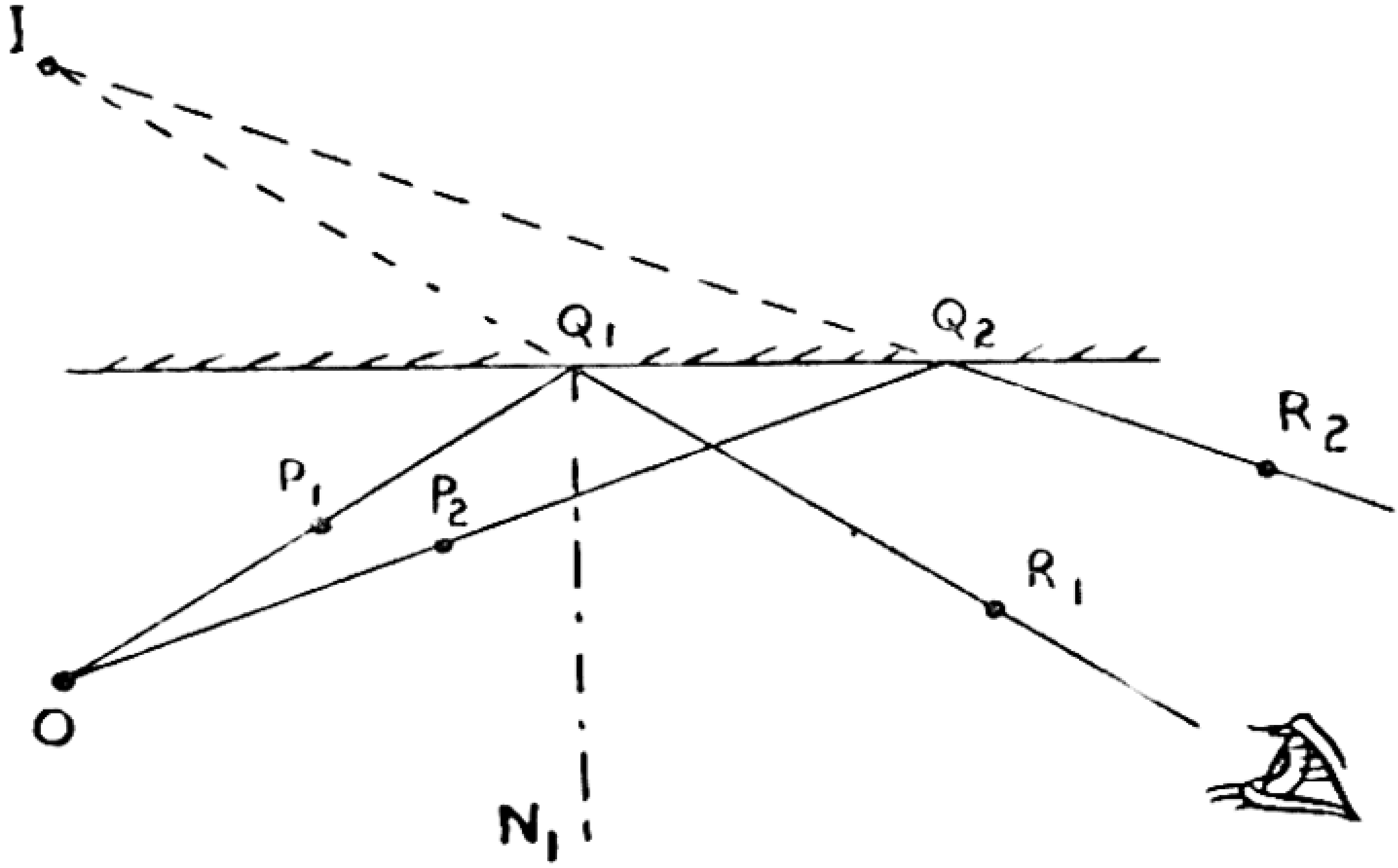


FIG. 52.—IMAGE IN PLANE MIRROR.

looking in the direction of the reflected ray (reversed), and you can fix its bearings by putting a third pin at R_1. Join Q_1R_1 by a line on the paper and also mark the position of the front edge of the mirror by another line. At Q_1 draw a line Q_1N_1 at right angles to the edge of the mirror to act as *normal*. Measure the angle of incidence $\angle OQ_1N_1$ and the angle of reflection $\angle N_1Q_1R_1$ and verify that these are equal (compare p. 108).

Replace the mirror; remove all the pins except O. Put in new marker pins P_2 and R_2 to mark another incident ray and its corresponding reflected ray, and the normal at the new point of incidence Q_2. Verify that $\angle OQ_2N_2 = \angle R_2Q_2N_2$. (Note that an alternative method is available for drawing a normal to the mirror at a point such as Q in

the absence of a set-square, if one puts a pin N in such a position that, looking in the mirror along the direction NQ, the pin at N seems to cover its own image. Join N to Q. The student should appreciate that in this method of drawing a normal the angles of incidence and reflection are both zero.)

Position of the Image in a Plane Mirror

1. By Geometrical Construction.—Since in Fig. 52 the reflected rays Q_1R_1 and Q_2R_2 are both seen to be coming from the image of the pin O in the mirror, removal of the latter and the production of these two lines until they meet will locate the image, I. If the line joining IO be drawn and measured in its relation to the line marking the mirror surface (or its continuation, if this be necessary), it will be found that the latter bisects IO at right angles. (The reader versed in geometry can prove for himself without measurement that this must follow from the equality of the angles of incidence and reflection.) So we may state:

In a plane mirror, the image is found on the normal to the mirror passing through the object, and as far behind the mirror as the object is in front.

2. By Parallax.—An alternative method of locating an image is based on the principle of parallax, which concerns the method by which we judge which of two objects is the farthest away from us.

When we look at two distant lamp-posts down a street, our estimate of which is the nearer of the two and how far they are apart is based on (*a*) which looks larger, (*b*) which seems to have more houses or more paving-stones between us and it. If, however, all the surroundings were blacked out, leaving just the two isolated lamp-posts, (*b*) would fail us, so also would (*a*) if we were comparing the distance of two things not known to be of the same height—for example, a lamp-post and a tree. The astronomer is in this position; the stars at night are dotted about a black firmament with nothing to guide him as to which is far and which is near. But as the earth rotates he sees a pair of stars from a different aspect and from the change in their relative position he can

judge which is the nearer. Thus if A and B (Fig. 53) are regarded from the position E_1, one is seen behind the other; move the eye to the right at E_2, and B appears to move to the right, relative to A; move the eye to the left, and B now appears to the left of A. Thus *in whichever direction the eye is moved the star farther away appears to move with the eye and the one nearer in the opposite direction.* If, however, B is exactly on top of A, they will always appear to 'cover' one another, no matter from what direction they are regarded. Under these circumstances we say that there is *no parallax* between A and B.

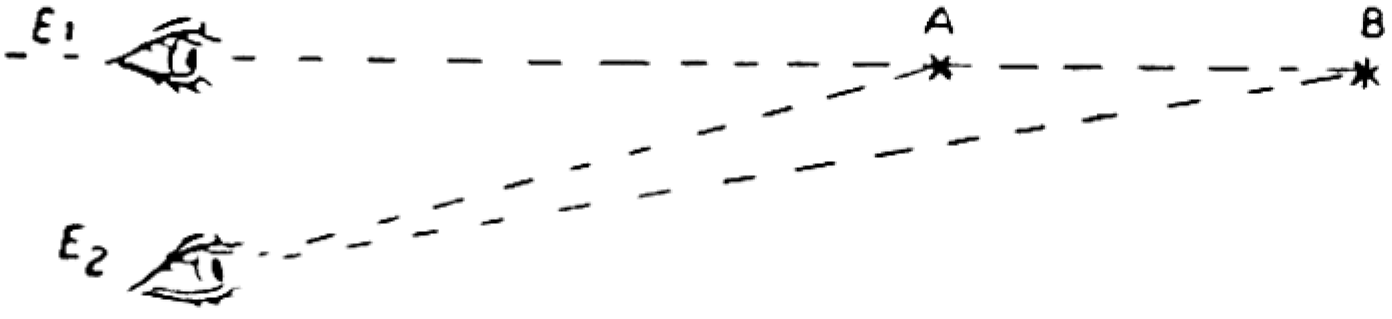


FIG. 53.—PARALLAX.

Applying this to our plane mirror, we may (with the object pin and the mirror in position) stick in a tall pin *behind* the mirror where we think the image of O is. Now look into the mirror at the image I and *over the top of it* at the marker pin—and observe whether one covers the other exactly and *continues to do so* as the head is moved to and fro. If not, then by the above rule we can decide whether our new marker pin is farther off than the image or nearer to us, and adjust its position accordingly. When there is no parallax between the tall pin and the image of O, the former should lie on the place which we found for the image by our previous drawing.

Size of the Image in a Plane Mirror

To estimate the size of an image relative to the object, it is useful to employ for object a short length of a wooden scale, say, 1 inch long, mounted so that it can stand on a level with the mirror with its scale *horizontal* and facing the mirror (see Fig. 51) or, alternatively, a piece of squared paper of about the same size pasted on a small block of wood.

Set this up facing the mirror in about the same position as O (Fig. 52). By the parallax method find with a pin the position of the image of one end of the little scale, and with a second pin the position of the other end. Measure the distance separating these two marker pins. It should be found that this is equal to the distance—namely, 1 inch—between the ends of the scale which forms the object. Thus we have proved that *in a plane mirror, the image is the same size as the object.*

The Curved Mirror

We can now proceed to apply these methods of experiment to investigating the position and nature of the images formed in a curved reflecting surface, commencing with the one which presents a concave surface to the eye.

The first thing we must do is to find the centre of the circle of which the surface forms an arc. Now, the laws of reflection will still apply to the curved surface, and as it is a property of the circle that any line drawn from the centre will meet the surface at right angles, the centre of the circle is that point towards which every ray sent out from an object there returns. In fact, if you set a pin at the centre of curvature of the mirror and look from it towards the mirror, it will always cover its own image, no matter in which direction you look. When the object is so placed, the image, being the point to which all the reflected rays converge, is actually formed on the object, so that there is no parallax between the object and its image. With a pin then find a position in front of the mirror such that, as you move your head from side to side, the object and its image always keep together.

Since the image is real, in this case we may employ our pea-lamp with a piece of white card placed just beside, or set—like a collar—over the bulb, until we get a sharp image formed alongside the lamp on the card. Measure the distance of this point from the surface of the mirror. This is the radius of curvature of the mirror.

It will be noted that the image is the same size as the object, but upside down.

Now move the object (pin or lamp) farther from the mirror and search for the image (with another pin or a little screen) nearer to the mirror than the centre of curvature.

There are certain precautions to be taken in doing this.

1. Lamp and Screen Method.—If a large screen which cuts off a good deal of the light from the lamp that would otherwise strike the mirror is used, one cannot, of course, expect to see the image. Use a small screen, and place the lamp a little to one side of the axis of the mirror (*an imaginary line coming out of the middle of the mirror and passing through the centre of curvature*). The image may then be looked for with the screen a little to the other side of the axis.

2. Parallax Method.—It is important not to confuse the image of the object which one is looking for with that of the pin which one is using as marker. A little flag of white paper may be attached to the object pin, and the other bare pin used to locate the image of the flag. Also, remember that you cannot see behind your head! Stand so far back from the mirror that the image is formed always in front of the eye when you are looking for it.

As the object is moved farther and farther away, the image moves slowly towards a point which is half-way in towards the mirror from the centre of curvature. *The position at which the image is formed in an optical system when the object is at infinity is called the* (principal) focus of the system; the word 'principal' is usually, however, omitted. *The distance from the focus to the mirror is called the* focal length. In a curved mirror *the focal length is half the radius of curvature.* When an object is at infinity the rays which strike the mirror form a parallel bundle, which after reflection then pass through the focus.

We have thus arrived at two rules for the graphical construction of images which are quite general, namely:

(1) A ray passing through the centre of curvature retraces its path.

(2) A ray passing parallel to the axis, passes after reflection through the focus.

Fig. 54 (*a*) shows how these rules are applied in the case

we have been discussing—*concave mirror ; object beyond the centre of curvature.* The arrow represents the object, at right angles to the axis, which may be the little object scale. We draw a ray from the head of the arrow parallel to the axis (shown by a chain line). Join QF (F being the focus) and produce it. The image of the head of the arrow lies somewhere on this line. Next join OR (R being the centre of curvature) and produce it to the mirror. The image of

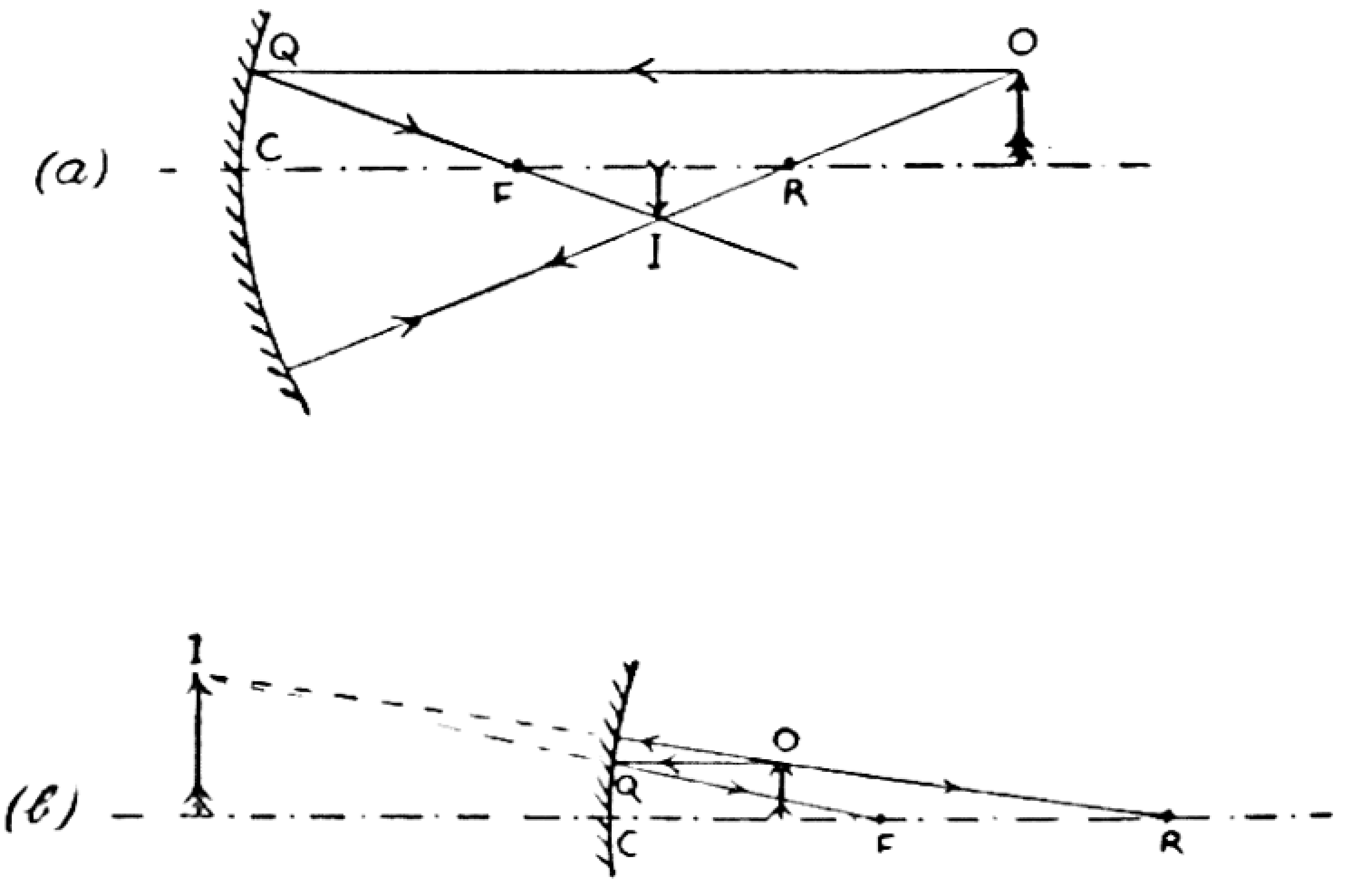


FIG. 54.—PATHS OF RAYS FROM CONCAVE MIRROR.

the arrow-head lies on this line (reversed). It is therefore at I the junction of OR and QF, produced. The tail of the arrow is on the axis just above it. In this case, then, *the image is inverted and smaller than the object.* Verify this construction by using the little scale as object and two pins to mark each end of the image without parallax, as in the plane mirror. Note that a third ray could have helped us to find I by joining O to C—C is where the axis meets the mirror surface—and then making an angle of reflection equal to that of incidence ($\angle OCR$) on the opposite side of the axis. This would have given us the reflected ray CI.

Since all rays may be reversed without detriment to the argument, if we place the *object* at *I*, *between the focus and the centre of curvature*, we get a *magnified inverted image* at *O*, beyond the centre of curvature.

There is one case which this figure will not cover—namely, when the *object* lies *inside the focal length*. This case is drawn in Fig. 54 (*b*). If we proceed as before—join *OR* and draw a ray *OQ* parallel to the axis to be reflected along *QF*—we find that *OR* and *QF* meet behind the mirror at *I*. So the image is *virtual*, as in a plane mirror, and its position can only be found, apart from drawing, by the method of no parallax. *The image is the right way up and larger than the object.*

The student should verify each of these cases by experiment with the pins and, except in the last case, by lamp and scale.

Convex Mirror

The student can now repeat these experiments with the mirror which is convex towards the source of light and verify his findings by a similar construction.

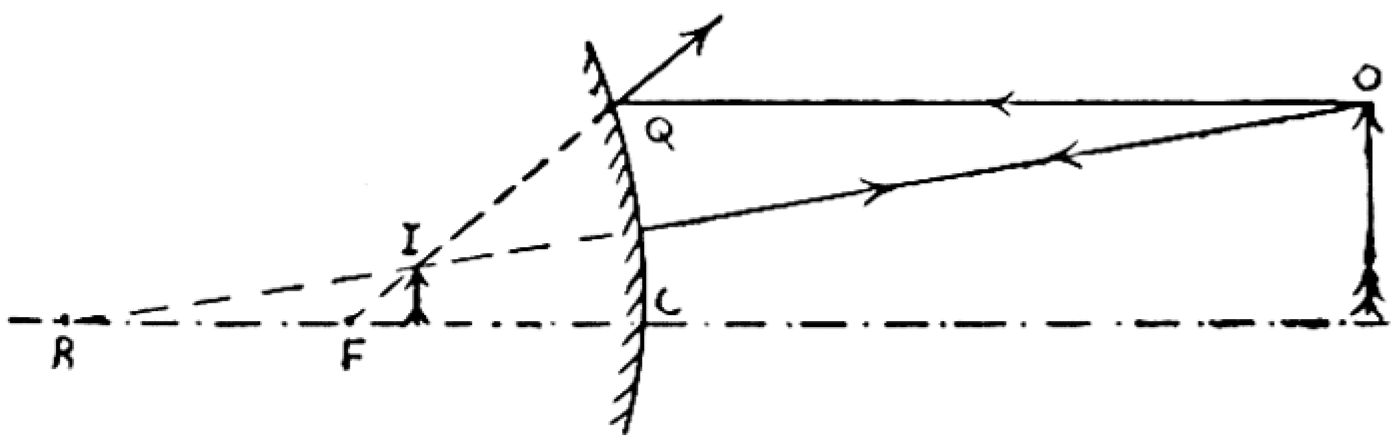


FIG. 55.—PATHS OF RAYS FROM CONVEX MIRROR.

The centre of curvature, *R*, now lies behind the mirror. If a parallel bundle of rays strike its surface, coming from an object at infinity, they will, after reflection, appear to *diverge from* the focus, which is also behind the mirror half-way in from the centre of curvature. Rays drawn towards the latter will, as before, retrace their paths. Thus, with an object at *O*, *the image will be virtual and diminished*, the

rays appearing to come from *I*. This will be so wherever *O* is placed so that there is only the one case to illustrate (Fig. 55).

It is possible to reverse the process by using a virtual object at *I* and getting a real image at *O* on a screen. The method of doing this will be described later (p. 163).

Formula for Mirrors

If a number of corresponding *object distances* (u) = distance between object and mirror—and *image distances* (v) = distance between image and mirror—are obtained for a mirror of known focal length (f) = half the radius of curvature, they will be found to fit the following formula:

Reciprocal of image distance added to reciprocal of object distance equals reciprocal of focal length

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

provided the following conventions as to the signs to be attached to numerical values are adhered to:

All distances are to be measured from the surface of the mirror towards the point in question; if, in doing so, you travel *against* the incident light, give this distance a *positive* sign; if *with* the incident light, a *negative*. This amounts to making lengths in front of a mirror positive, those behind, negative. It also makes the focal length of a concave mirror positive, that of a convex mirror, negative.

Example 1.—An object is on the axis of a concave mirror at 30 cm. distance. The mirror has a radius of curvature of 20 cm. Where is the image?

Put $f = +\ 10$ cm.; $u = +\ 30$ cm.; then

$$\frac{1}{v} + \frac{1}{30} = \frac{1}{10};\ \frac{1}{v} = +\frac{2}{30};\ v = +\ 15 \text{ cm.}$$

The image is in front of the mirror, 15 cm. from its surface.

Example 2.—The mirror is now replaced by a convex one of the same focal length. Where is the image?

Put $\quad f = -10$ cm.; $u = +30$ cm. as before

$$\frac{1}{v} + \frac{1}{30} = -\frac{1}{10}; \ \frac{1}{v} = -\frac{4}{30}; \ v = -7{\cdot}5 \text{ cm.}$$

The image is behind the mirror, 7·5 cm. from its surface.

The relative sizes of object and image produced by a mirror may also be expressed by a simple formula:

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{Image Distance}}{\text{Object Distance}} = \frac{v}{u}.$$

Thus, in Example 1 the image is half the size of the object; in Example 2 it is one-quarter. (This may be verified by experiment and by simple geometry applied to Figs. 54 and 55.) In a plane mirror, or in a curved mirror in which the object lies at the centre of curvature, the image is the same size as the object.

Mirrors of Large Aperture. Caustic Curve.

The rules which we have given for curved mirrors apply only if the mirror has a small width—that is, does not extend far on either side of the axis relative to the object and image distances. In fact, when an optical system consisting of spherical mirrors or lenses has a wide aperture (this being the term used for *width perpendicular to the axis*) a point source on the axis does not produce a point image, but gives rise to a blur. We shall illustrate this by reference to a concave mirror of wide aperture. One of a focal length of a few inches and several inches wide is suitable—for example, a strip of polished tinplate or stainless steel bent into a semicircle of 3 inches radius and stood on a sheet of paper will serve for this experiment.

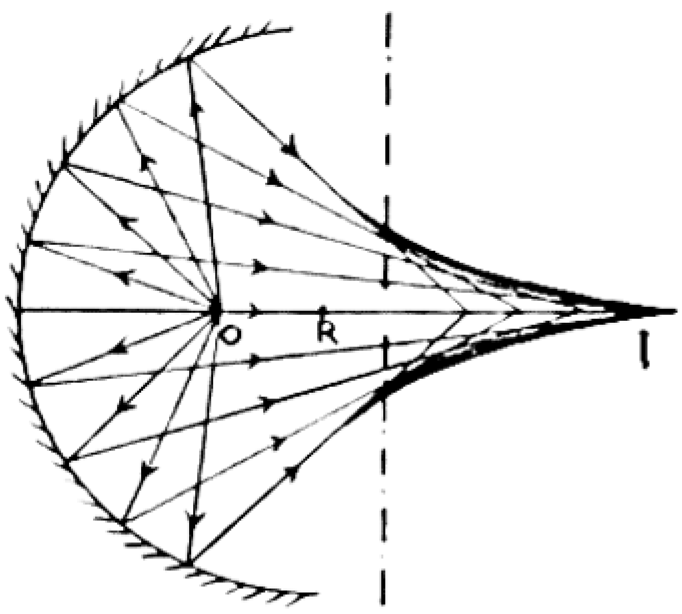


FIG. 56.—CAUSTIC CURVE BY REFLECTION

Place the pin object O on the axis inside the centre of curvature R, and with the aid of marker pins P and Q trace a number of rays up to the mirror surface with their corresponding reflected rays, including those which make quite a large angle with the line joining OR (Fig. 56). It will be noticed that as long as rays making a small angle to the axis only are included, the reflected rays converge as before on the point I, but rays more inclined strike the axis at points nearer to the mirror, the greater their inclination. These rays ' touch ' a curve, and if sufficient of these ray tangents are drawn, the two arms of the curve converging on I will be mapped out, as shown in Fig. 56.

This curve may actually be seen as two bright lines on the surface of the paper if the pin O be replaced by a brightly lit lamp. The student may then convince himself that it is actually the more inclined rays which are contributing to form the major part of the curve by pushing in two stops in the form of pieces of card from either side just in front of the mirror (chain lines in Fig. 56). When these are sufficiently close to leave a narrow chink the brightness will reduce itself to a single spot at I.

Because this curve can be seen, as it were, burnt into the paper, it is known as the caustic curve. It can also be seen on the surface of tea or coffee in a nearly full cup, when the sun or a lamp shines in over one edge.

CHAPTER XII

REFRACTION

Paths of Rays through a Parallel-sided Slab

For this experiment we place the glass slab (or rectangular tank filled with water) on a sheet of white paper with a pin *O* (Fig. 57) a few inches away from one side, to serve as

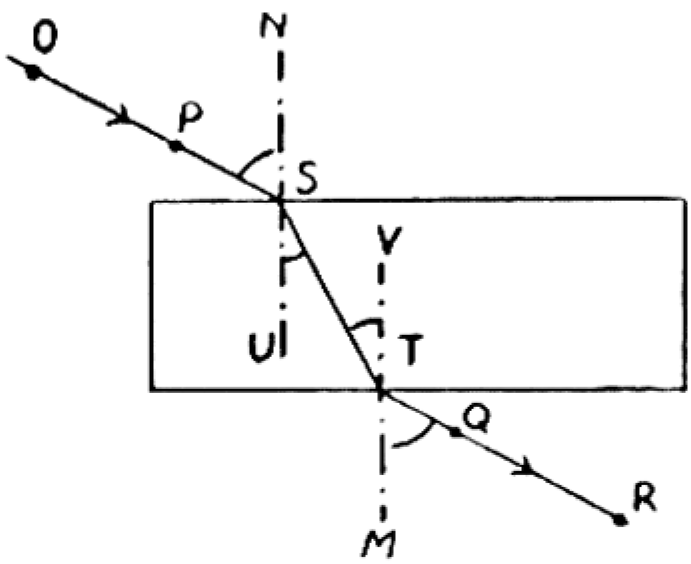


FIG. 57.—REFRACTION OF RAY THROUGH GLASS SLAB.

object. We shall now trace rays through the slab, by setting a marker pin *P* nearer to the surface and looking through the glass *from the other side*, until we see the images (by refraction) of *O* and *P* in a straight line. Set two pins *Q* and *R* on this side in line with the apparent position of *O* and *P*. Draw round the slab with a pencil and then remove the slab.

On drawing the lines *OP* and *QR* it will be seen that they are not continuous (as they looked when seen through the glass), but are parallel. Produce *OP* to meet the slab at *S*, and *RQ* to meet it at *T*. The line joining *S* and *T* must then represent the path of the ray inside the glass. Verify this by replacing the slab or cell and observing that, seen from *R*, the whole pencil-mark back to *O* appears to be straight.

Draw normals *USN* and *VTM* to the surface. Then $\angle OSN$ is the angle of incidence (i) and $\angle UST$ the angle of

refraction (r). By drawing or by reference to trigonometrical tables calculate the index of refraction (p. 109); namely, $\mu = \frac{\sin i}{\sin r}$, for the media air and glass, or air and water respectively.

Note that by the geometry of the figure $\angle VTS = \angle UST$ and $\angle RTM = \angle OSN$; in fact, we may regard R as object and consider the ray to pass along RQ through the glass and out on the other side in the direction of P.

Repeat the experiment with P in a different place so that the ratio $\sin i/\sin r$ remains the same. (Compare the laws of refraction, p. 109.)

Caustic Curve by Refraction

If the object pin O be placed right up against the glass surface, a series of emergent rays can be traced by putting pairs of pins Q_1R_1, Q_2,R_2, etc., into the paper on the other

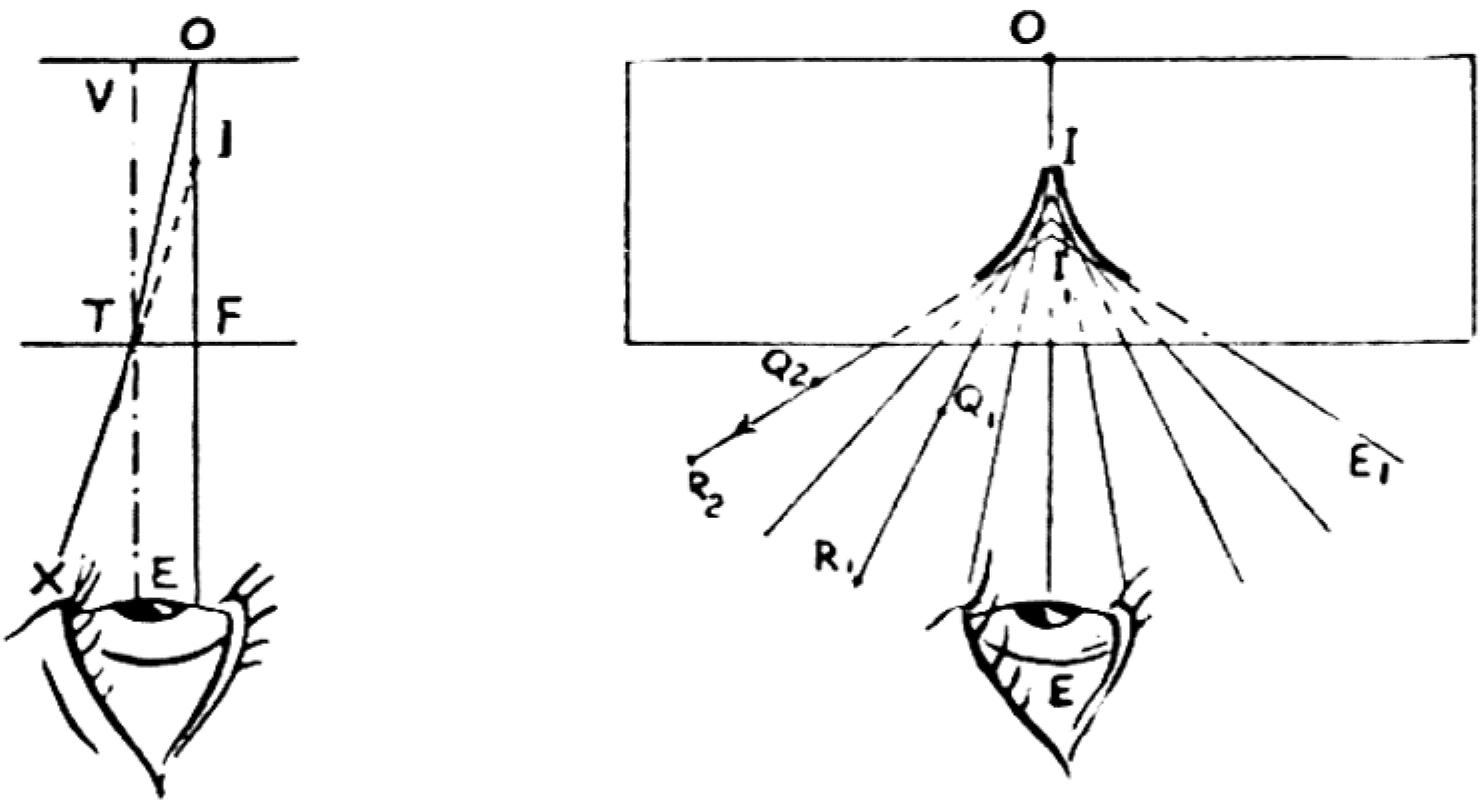


FIG. 58.—CAUSTIC CURVE BY REFRACTION.

side, so that they are seen in line with the image of O (Fig. 58). These lines are tangents to a curve, and if produced back will, in sufficient number, trace out the curve, which is similar to that formed by reflection in a concave mirror, and is called a caustic curve by refraction.

There is one important difference, however, The caustic formed by reflection has a real existence, as we have shown by the experiment in the tea-cup. The *caustic by refraction*, being merely the prolongation of emergent rays into the glass, *has no real existence*, the actual paths of the rays in the glass being such as OT. The construction of it is, nevertheless, more than an academic exercise, as the next section shows.

Real and Apparent Depth

As the eye looks obliquely into the glass or water from E, the image of the pin is seen at I, where the two extreme rays coming into the eye appear to diverge from. Looking down normally from E, the eye sees the image of O at I. Since O is at the far edge of the glass, the real thickness OT of glass (or water) is replaced to an observer with his eye at E by the apparent thickness IT. Light which has actually entered the eye by the route OTX, appears to come by the straight line ITX (Fig. 58; left-hand drawing).

Now $\angle ETX = \quad TV = \angle TIF = i$: $\angle OTV = \angle TOF = r$

so
$$\mu = \frac{\sin i}{\sin r} = \frac{\frac{TF}{TI}}{\frac{TF}{TO}} = \frac{TO}{TI}.$$

Now the direction XT is so nearly normal to the glass that we may consider it as the direction of practically perpendicular vision into the glass; then TO becomes the *real* thickness (or *depth*), FO and TI the *apparent* thickness (or *depth*), FI.

$$\text{Then } \frac{\text{real depth}}{\text{apparent depth}} = \begin{array}{c}\text{refractive index of the medium}\\ \text{in the slab (relative to air).}\end{array}$$

Verify from your construction that the ratio FO/FI gives the same answer for the refractive index of the glass (or water) to that which you got in the previous experiment.

That the bottom of a trough looks nearer, as one peers straight down into it, when it is filled with water than when it is empty is a matter of common observation. If one

looks more obliquely into the water over the edge (by standing back a little) it appears shallower still. This is explained by Fig. 58, since one is then, in effect, moving the eye from E to E_1 and the image accordingly goes from I to I_1 nearer to the surface.

As you walk beside a swimming-pool with a brick-lined bottom, a bulge seems to form in the bottom which travels with you, the top of the bulge being always just opposite to you as you walk. Think out for yourself why this is so.

Paths of Rays through a Prism

The prism in light experiments is a slab of glass or trough of liquid of which the cross-section is a triangle.

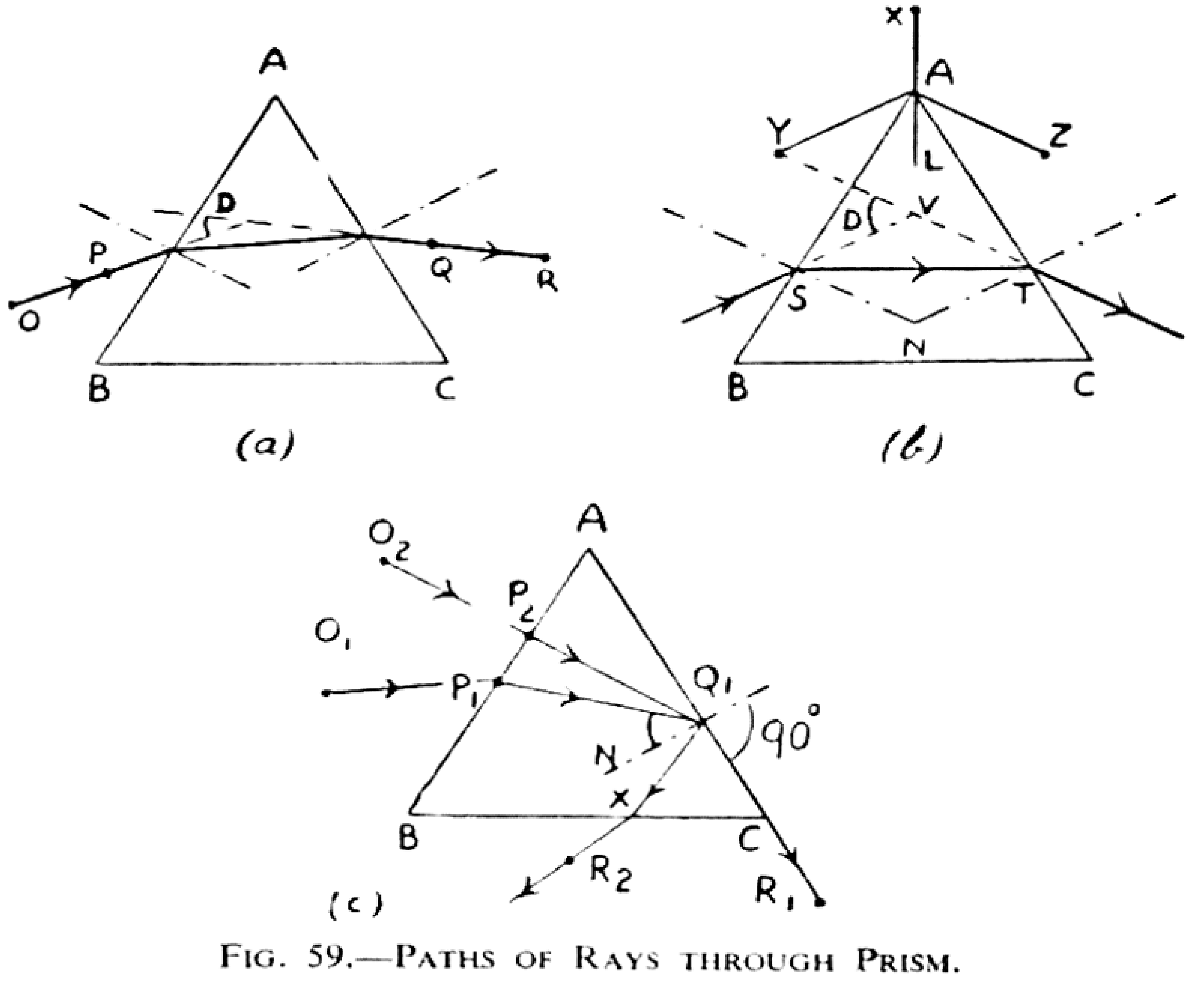


FIG. 59.—PATHS OF RAYS THROUGH PRISM.

Place the body with one of its triangular sides flat on a piece of paper on the table and set up as before two pins O

and P (Fig. 59 (a)) to define an incident ray falling obliquely on one side (AB). Looking in from the opposite side (AC), set two more pins Q and R in line with the images of O and P as seen from this side. Having joined OP and QR and produced them to meet the sides AB and AC (which should be marked), remove the prism and complete the path of the ray through the prism. *The angle between the incident and the emergent ray*, marked as D on the figure, is known as the deviation.

Repeat this for a number of angles of incidence and select the one for which the ray passes symmetrically through the prism (Fig. 59 (b))—that is, when the angle of emergence equals the angle of incidence. If the deviation is measured in each case, it will be found that it is least of all for this symmetrical case. It is therefore known as *the position of minimum deviation.*

Measurement of Angle of Prism.—Although the triangle forming the prism has, of course, three angles, it is always the angle A, the one which forms the vertex to the base formed by the path of the ray through the prism, which is called, in light experiments, the *angle of the prism.*

To measure this, set a pin X where the vertex A points towards it. Look in the face AB and set another pin Y to cover the corner A and the image of X *reflected* in the glass. Do the same with the pin Z on the other face AC. Remove the prism, joining AY, AZ and XA, and produce the latter to L.

Because, by the laws of reflection, XA and YA are equally inclined to the surface AB, $\angle LAB = \angle BAY$ and $\angle LAY =$ twice $\angle LAB$. Also $\angle LAZ =$ twice $\angle LAC$. Hence if $\angle YAZ$ be measured, it is double the angle A of the prism.

(Of course, the angle A can also be measured directly with a protractor from the trace BAC of the prism on the paper.)

Calculation of Refractive Index of the Material of a Prism

With the prism in the position of minimum deviation (D) and a knowledge of the angle (A) of the prism, the refractive

index of the glass of which it is formed, if solid, or of the liquid contents, if hollow, can be calculated from the formula:

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A}.$$

[In Fig. 59 (*b*), the incident and emergent rays are produced to meet at V; the normals at S and T meet at N. $\angle VSN = \angle VTN = i$; $\angle SVT = 180° - D$. Since $\angle NST = \angle NTS = r$; $\angle SNT = 180° - 2r$. Also, since $\angle NSA = \angle NTA = 90°$; $\angle SNT = 180° - A$; while, since $\angle VST = \angle VTS = (i - r)$; $D = 2\ (i - r)$; finally, then $r = \frac{1}{2}A$; $i = \frac{1}{2}(A + D)$].

Total Internal Reflection

Let us now trace a ray incident at a smaller angle to the normal (Fig. 59 (*c*)). As the angle of incidence is increased we have to look into the prism on the other side from a direction almost grazing the surface of the prism. Eventually, a ray O_1P_1 will be reached which strikes the other face through the glass at such an angle that it actually runs parallel to the surface on emergence along the direction Q_1R_1. If we retrace this ray along R_1Q_1 in air, we see that the angle of incidence (i) is 90° on this side, while P_1Q_1N is the angle of refraction (r) in the glass (or water). This last *angle*—namely, that *corresponding to emergence along the surface—is called the* critical angle. Since the sine of 90° is 1, we see that the *sine of the critical angle is equal to the refractive index* for the medium in question.

The reason for calling this the critical angle will be apparent if we increase the angle on the AB face a little, so that the ray enters along O_2P_2, and meets the other side at Q_1 at an angle greater than the critical angle. Under these circumstances it cannot emerge, but is reflected within the glass in the direction Q_1X_1 to emerge from the face BC. This is called total internal reflection.

A prism set so that rays entering it fall on the opposite face at an angle greater than the critical angle is sometimes used as an alternative to the plane mirror for turning light through an angle.

Total internal reflection may occur when light enters transparent bodies of other shapes than the prism. For

instance, when light enters a globe of glass or water at certain angles it may be totally reflected on the far side, because it strikes at a greater than the critical angle, and so re-emerges on the same side that it went in, but turned somewhat from its original direction. This occurs when sunlight enters water drops and gives rise to the rainbow, which we shall discuss again later.

Lenses. Derivation of a Lens from a Prism

If we imagine two thin prisms of, say, 5° angle set with their bases in contact, we have a system which will converge light passing through it from a source situated on the axis of the system, which is formed of a line passing through the junction of the bases at right angles to the triangular sections of the two prisms (Fig. 60*a*). Now, instead of the abrupt

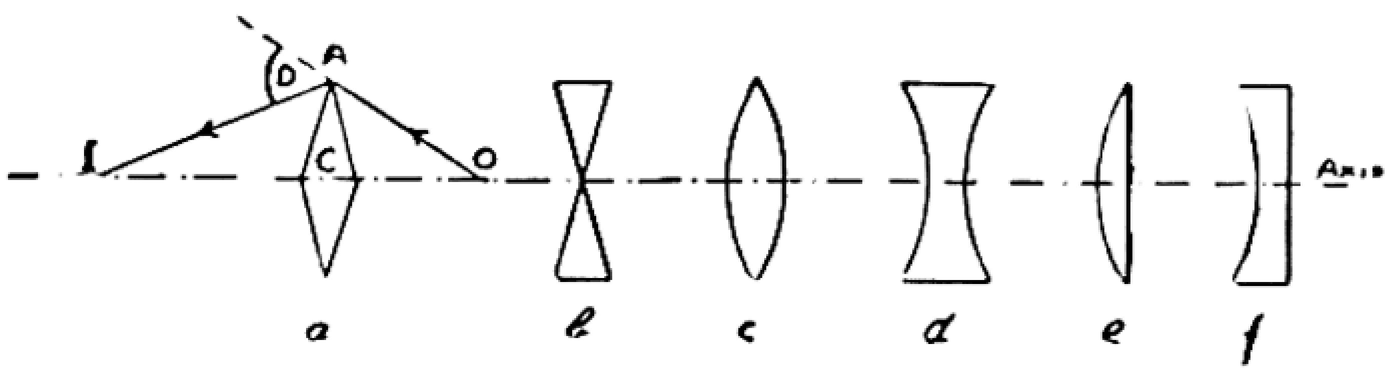


FIG. 60.—DEVELOPMENT OF LENSES.

change in direction of the sides of the double prism where the bases join, let us run two smooth curves in the form of circular arcs from vertex to vertex, and we have the section of a converging lens (Fig. 60*c*).

In the same way, two prisms set with their vertices touching will diverge light from a point on their common axis (Fig. 60*b*). Replacing the prisms by a piece of glass with concave arcs in place of the inward-bending straight lines, we have a diverging lens (Fig. 60*d*).

If the lens is symmetrical in form in all directions round its mid-point (like a spherical mirror) it is a *spherical lens*. If, however, the section is the same all about a plane through the glass, it is a *cylindrical lens*. In what follows we shall, unless special mention is made of the cylindrical lens, refer exclusively to the more common spherical lens.

Usually the two surfaces have the same curvature, though one may be plane (Fig. 60, *e*, *f*). It is usually a sufficient test of the difference between a convergent and a divergent lens to feel its thickness between the thumb and fore-finger. The convergent lens is thicker in the middle than at its edges; the divergent lens is thinner in the middle.

Formula for Lenses

We can derive the formula for lenses corresponding to that already given for mirrors by making use of the formula for the deviation produced in a prism (p. 156). In a thin prism A is so small, and so therefore is D, that the sines of these angles may be replaced by the angles themselves—namely, we may write:

$$\mu = \frac{\frac{D + A}{2}}{\frac{A}{2}} = \frac{D}{A} + 1$$

or

$$D = A(\mu - 1)$$

Now let O be an object shining on to one of the prisms in a beam, circumscribed by OA and OC (Fig. 60*a*), which after refraction in the prism emerges to converge on I. We have marked the deviation D as the angle between the extreme incident ray $= OA$ (produced) and the corresponding emergent ray, AI; also $OC = u$, the object distance, and $IC = v$, the image distance.

Now $\angle D = \angle AOC + \angle AIC$, and since these two latter angles are all small, we can replace them by their tangents $\frac{AC}{OC}$ and $\frac{AC}{IC}$, respectively; so

$$A(\mu - 1) = \frac{AC}{v} + \frac{AC}{u}.$$

Now, considering only possible variations of v and u with the same lens, all the other quantities are constant, so that $\frac{1}{v} + \frac{1}{u}$ is a constant. Now, we shall define the focus as that

position of the image corresponding to a parallel beam of light, or in other words to O being at infinity. $\frac{1}{u}$ is then zero, and $\frac{1}{v}$ equals $\frac{1}{f}$, if we put the constant as $\frac{1}{f}$, f being the focal length in the same sense as for a mirror.

However, if we accept the sign convention already adopted for mirrors (p. 148), we must call v in the figure negative and u positive, for we are going against the direction of the light in measuring from C to O and with it from O to I. Thus to safeguard the relationship that the sum of the angles at O and I is constant, and remembering that we should have to write the value of $\angle AOC$ or its tangent, $\frac{AC}{OC} = \frac{AC}{u}$, we finally put our lens formula in the form:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

Reciprocal of image distance minus reciprocal of object distance = reciprocal of focal length.

The reciprocal of the focal length is known as the focal power; lenses used by opticians are usually specified in terms of the latter quantity on the basis of the dioptre. A lens has a power of one *dioptre* when its focal length is one metre. Note that a lens has two foci, one on each side.

The Convergent Lens

We shall now mount our convergent lens in its metal holder, set it up on a sheet of white paper, and begin to trace rays through it, checking our observations by a drawing construction, using the two rules which result from our definition of the focus, namely:

1. A ray which passes into the convergent lens parallel to the axis, emerges so as to pass through the focus *on the other side.*

2. A ray which passes into the lens after going through its central point meets two surfaces which are parallel, and so emerges with direction unchanged (see p. 146).

To find the focal length of a convergent lens roughly, hold it some distance from a window and holding a card on the other side of the lens obtain a clear image (which, you will note, is upside down) of the window on the card. The window is so far off that the object distance may be taken as infinity, so the card is then at the focal length from the lens. Replacing the lens on the table, set up a pin, *O*, beyond the focus (Fig. 61 (*a*)), and with a marker pin on the

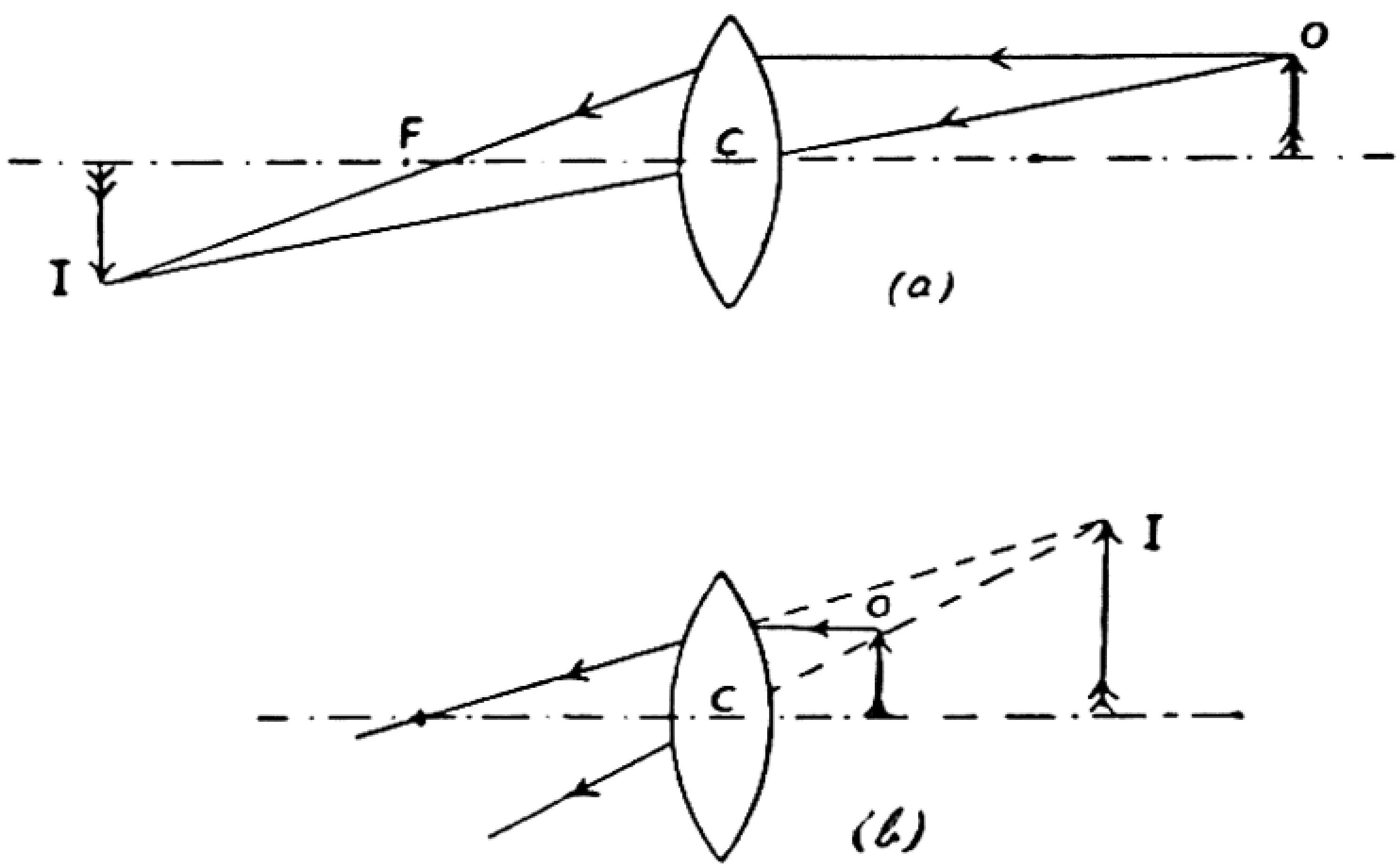


FIG. 61.—RAYS IN CONVERGENT LENS.

other side of the lens look for the image of *O* seen through the lens and move the marker pin to *I*, where there is no parallax between it and the image of *O*. It will be noted that the image is beyond the focus on the far side. We verify this by taking an object of finite size at *O*, perpendicular to the axis, and constructing the image by drawing (Fig. 61 (*a*)), using the two rules just given.

Verify that the rule that we gave for mirrors

$$\frac{\text{Size of Image}}{\text{Size of Object}} = \frac{\text{Image distance}}{\text{Object distance}} \text{ still holds.}$$

Since the image is real in this case, we may put our

pea-lamp at O and search for the sharp image on a card at I.

When the pin O *is placed at less than the focal distance from the lens, the image is the same way up and on the same side*, and must be sought by looking through from the far side with the marker pin held on the same side as the object pin. The student is likely to be confused if he can see both pins through the lens, so that it is better to hold the marker pin in the hand or in a clamp, so that it can only be seen *over the top of* the lens while the object pin, O, can only be seen *through* the lens. Adjust to get no parallax between the image and the marker.

The corresponding drawing is shown in Fig. 61 (*b*). As the image is virtual in this case, the lamp and card method fails.

(The student should note that in a mirror a real image is on the same side as the source of light, while a virtual image is on the opposite side; in a lens a real image is on the opposite side to the source, while a virtual image is on the same side. This is because reflected rays have a real existence only on the side of the source of light towards which they are reflected; refracted rays have a real existence only after the light has passed into and through the lens.)

The Divergent Lens

Since this lens makes a beam of light diverge more after passing through it, our two rules of construction become:

1. A ray which passes into the divergent lens parallel to the axis emerges as though it has passed through the focus on the *same* side.
2. A ray which passes into the lens at its central point emerges undeviated.

Since *a divergent lens gives a virtual image of a real object*, it is not possible to focus an image of the distant window on a card; the focal length must be found by calculation.

Set up a pin O before the lens and looking through the other side use another pin to mark the virtual image I in the same way as that used for the convergent lens with virtual image. The prediction of the size and position of the

image follows by application of the two rules just given (see Fig. 62).

Application of the Lens Formula

Example 1.—An object is placed on the axis of a convergent lens at a distance of 5 cm. from its surface. A real image is formed at 10 cm. from the lens on the other side. Calculate the focal length of the lens (Fig. 61 (*a*)).

$$u = +5;\ v = -10;$$
$$\frac{1}{f} = \frac{1}{-10} - \frac{1}{+5} = \frac{-3}{10};\ f = -3\tfrac{1}{3}\text{ cm.}$$

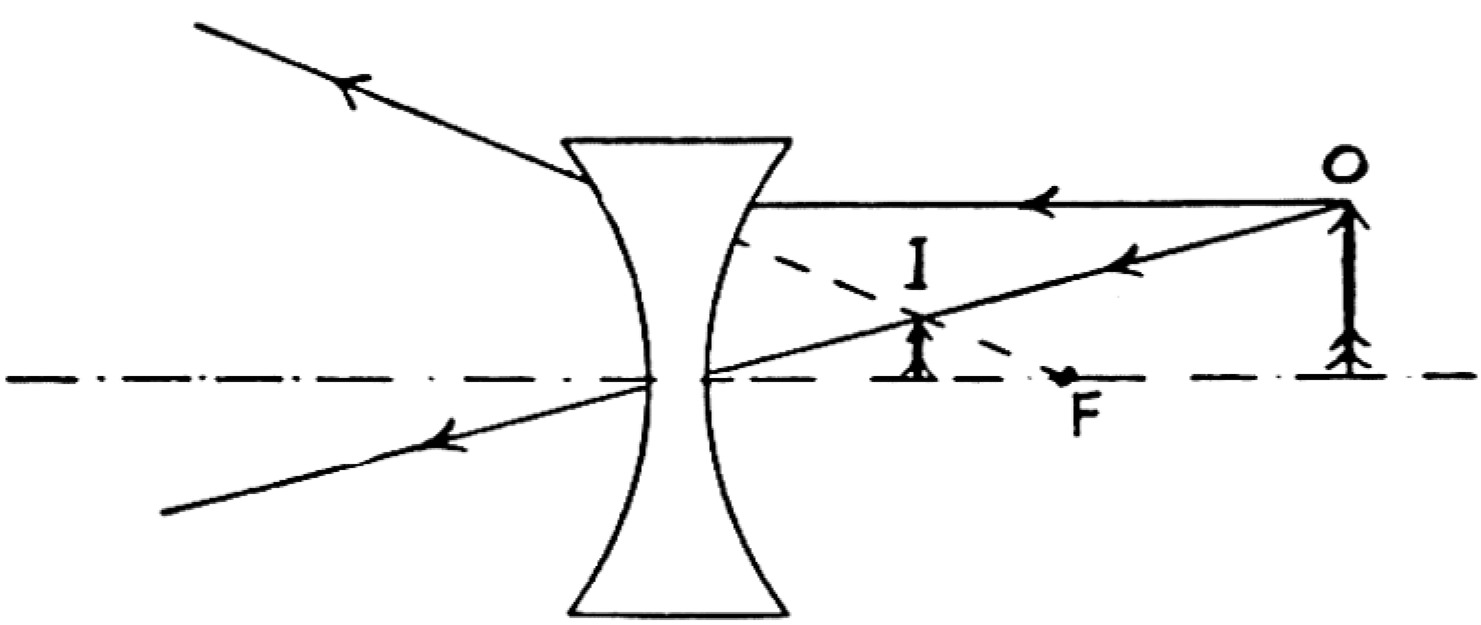


FIG. 62.—RAYS IN DIVERGENT LENS.

The object is now moved up to 2 cm. from the lens; where is the image ? (Fig. 61 (*b*)).

$$u = +2;\ f = 3\tfrac{1}{3}$$
$$\frac{-3}{10} = \frac{1}{v} - \frac{1}{+2}$$
$$\frac{1}{v} = \frac{-3}{10} + \frac{1}{2} = \frac{+1}{5}.$$

Answer: the image is virtual, 5 cm. from the lens on the same side as the object.

Example 2.—An object is placed on the axis of a divergent lens 5 cm. from it and an image is formed 1 cm. from

the lens on the same side. What is the focal length of the lens? (Fig. 62).

$$u = +5, v = +1$$

$$\frac{1}{f} = \frac{1}{1} - \frac{1}{5} = \frac{4}{5};\ f = +1\tfrac{1}{4}\text{ cm.}$$

It will be noted that, with the sign convention here adopted, *the focal length of a convergent lens is negative, that of a divergent lens is positive.*

The Use of a Lens to form a Virtual Obiect

We have explained that the difficulty that prevents us in the ordinary way getting a real image from an object both with the convex mirror and the divergent lens is the natural tendency of these systems to make a parallel beam diverge after reflection or refraction, respectively. If, however, we could bring a strongly convergent beam to be incident upon them, the net result would be that it would still converge, though naturally not so strongly as at first. What happens may be expressed by the formula:

Strong Convergence plus Moderate Divergence equals Weak Convergence

instead of:

Parallel Beam plus Moderate Divergence equals Moderate Divergence.

This is known as the method of the virtual object, the latter being at the point towards which rays would converge

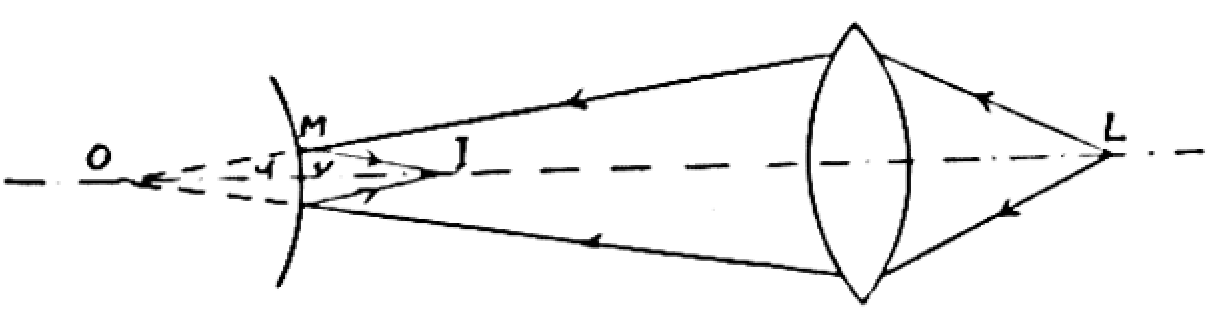


FIG. 63.—VIRTUAL OBJECT FOR CONVEX MIRROR.

if not interrupted by the convex mirror or divergent lens, as the case may be. Fig. 63 shows this method applied to the convex mirror. Light coming from the pea-lamp at L

would, if the lens were alone, be caught on the screen at O, but the mirror is put in the way at M, facing towards the lamp. This causes the rays which would have converged at O to be reflected forward to form an image at I, which is real. It may be caught on a screen, if the axis of M is tilted a little with respect to that of L, so that the image is a little to one side, where the screen may be set without interrupting the light beam from L towards O.

(Note how u and v are marked on the figure, and that in a numerical example u would be negative and v positive.)

Combinations of Lenses

Sometimes pairs of lenses are used in close contact. In such a case the optical effect is calculated from the rule (with due regard for signs):

Focal power of combination equals sum of focal powers of components.

This gives us another means of finding the focal length of a divergent lens, for if we can combine it with a more powerful convergent lens and determine, by getting a real image, the focal length of the latter alone and of the—still convergent—combination, we can calculate the focal length of the divergent lens.

Example.—A convergent lens has a focal length of 8 cm.; when placed in contact with a divergent lens, the combination is found to have a focal length of 20 cm. What is the focal length of the divergent lens?

Let the latter be $+f$; focal power $\frac{1}{f}$.

For the convergent lens; focal length -8; focal power $\frac{1}{-8}$.

For the combination; focal length -20; focal power $\frac{1}{-20}$.

Therefore
$$\frac{1}{-20} = \frac{1}{-8} + \frac{1}{f}$$
$$\frac{1}{f} = \frac{3}{40};\ f = 13\tfrac{1}{3}\text{ cm.}$$

CHAPTER XIII

OPTICAL INSTRUMENTS

Instruments Employing a Single Convergent Lens

The convergent lens may, as we have seen, be used either to give a real or a virtual image, according as the object lies outside or within the focal length.

With the first of these positions it is used in the camera for obtaining a pictorial record of the object in the form of a chemically developed image on a sensitized film or plate. Light has the property of acting chemically on certain substances, notably of depositing silver from certain of its salts. In the original photograph a black silver deposit appeared on the bright parts of the image, while the salt remained unaffected in the dull parts, and could be dissolved off by washing. This gave the 'negative' with the light parts black; this was then turned into the 'positive' by a fresh exposure to the light of the dried negative, closely backed with another sensitized plate.

As the camera may be called upon to deal with an object at various distances—for example, a view 'at infinity' or a close-up portrait—it is necessary to be able to 'focus' the image on the plate or film.

This can be done in two ways. 1. The lens is mounted on a rack so that it can be moved forward and backward, an extensible bellows keeping the box between lens and plate light-tight. The photographer looks at a viewing screen of frosted glass which is substituted for the sensitized plate until he gets a sharp image.

2. The lens is at a fixed distance from the screen in a 'box-camera', but some adjustment of the effective focal length of the lens is possible by means of a stop, which is a circular opening of variable diameter to cut off light from the outer rim of the lens, more or less.

To see how this device acts, refer back to Fig. 56 (p. 149). There we noticed that only if the centre portion of the concave mirror were exposed to the light could we get a sharp

image at I of a point source at O. If light is allowed to penetrate to the outer rim of the lens, the image (*a*) becomes blurred, and (*b*) the centre of the extended image (or blur) moves nearer to the mirror. Thus we can, at the expense of some loss of definition or clearness, change the position of the image a little, while keeping object and mirror at the same place. The same thing happens with a lens, except that it is the caustic curve formed by refraction (p. 152) which is now responsible for the trick. Also, in the box-camera we keep the image in a fixed position on the screen, but use the stop to accommodate the system for change in position of the object.

One disadvantage of the second method, apart from blurring of the image if the stop is opened out too far, is the loss of light and therefore of sensitivity to short-time exposures at the other extreme, when the stop aperture is nearly closed.

Great strides have been made in the last few years in fashioning wide-angle lenses—that is, lenses which can be used unstopped, with a large aperture—and yet not suffer from the defect of producing a blurred image. This can be done in two ways: (1) by fashioning it so that the outer rim of the lens is made of glass having a different refractive index, or (2) by altering the curvature of the surfaces so that they are no longer exactly spherical. By either or both of these devices, rays which strike the outer portions of the lens at a steep angle to the axis may be brought to their proper place on the image, which is then undistorted. Strictly, the compensation is only correct for one image distance, usually that at the focal length, but then such a camera will be mainly used for distant objects.*

The camera obscura (literally 'dark chamber') is a larger edition of the hand camera in which the distance from lens to screen amounts to several feet. The darkened chamber separating them is sufficiently large for several persons to enter and see whatever image is formed on the white screen. The axis of the system is usually vertical, with the lens laid

* For further information, see 'Teach Yourself Photography' in this series.

horizontally over a hole in the roof. If it is desired to look around, a mirror set at 45° to the horizontal is mounted above the lens, and is capable of rotation about a vertical axis, so as to catch light coming from any desired direction. Though the camera obscura is now classed as harmless entertainment, it has a serious use in the 'periscope' of a submarine.

In the projection lantern the set-up of the camera is reversed. A glass positive (photograph or drawing) is

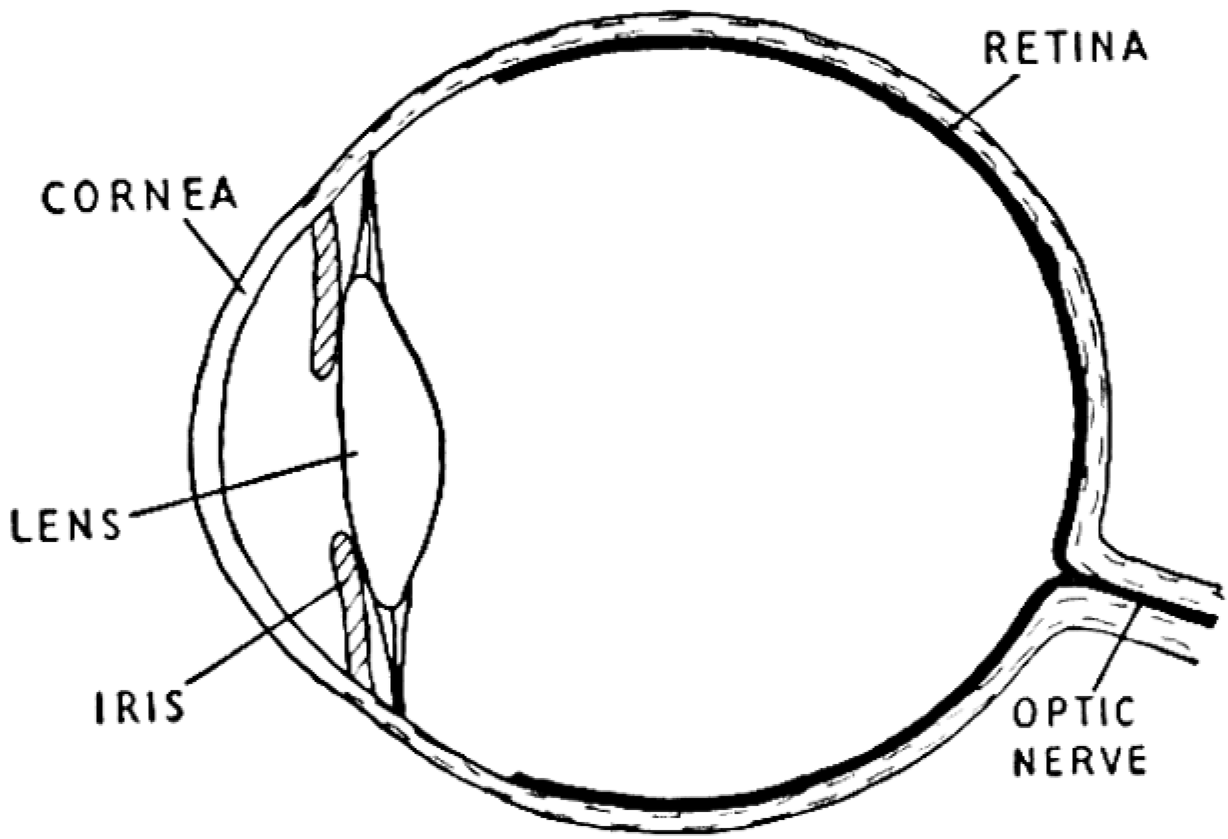


FIG. 64.—THE EYE.

illuminated by a powerful source training a parallel beam upon it. This acts as object, and it is placed near (but outside) the focus of a converging lens, so that an enlarged image is formed upon a white screen. From Fig. 61 it is obvious that the lantern slide must be inserted in the projector upside down, if the image on the screen is to be the same way up as in the original scene which is being reproduced.

The eye (Fig. 64 shows it in section) is a miniature camera. The convergent lens is at the front, the screen or retina at the back of the eyeball. The stop or iris

is in front of the lens. The medium between, however, is not air, but a liquid resembling water. The lens is normally 'focused for infinity', so that distant objects form a diminished image upside down on the retina, where it affects the optic nerve or nerves which convey the impression to the brain. As all images on the retina are inverted in this way, we are not mentally aware of this inversion.

With a lens of fixed focal length, as we have explained in connection with a camera, the image would be blurred when the object was brought closer to the eye, though some adjustment of its position relative to the retina would be possible by partial shutting of the iris, just as with the stop on a camera. The main function of the iris, however, is to cut down the amount of light entering the eye in brilliant illumination—this is well shown by the iris of a cat's eye, which is nearly shut by day, but wide open at night—and the retention of the image on the retina as the object approaches is effected by a process not possible in a glass lens—namely, a change in the focal length. This change, called *accommodation*, takes place in this wise. A muscle round the rim of the lens compresses it, making it more bulbous, and so more convergent. This shortening of the focal length can take place, in the normal human eye, until the object is as close as 10 inches from the eye. Nearer objects cannot be seen clearly without the aid of a spectacle lens, such as a watchmaker uses.

Not infrequently the lens of the eye in the rest position—that is, when the muscle is not pulling on it—is not of a suitable shape to focus a distant object precisely on the retina. If the focal length is too short—that is, the lens too convergent—a parallel beam of light falling on the eye will come to a focus before the retina. Without using accommodation, the persons can only see clearly if the incident beam is slightly divergent—that is, if it comes from an object 'nearer than infinity'. With normal accommodation his visual range will lie between this far point and some point nearer than the usual 10 inches, and he is called *short-sighted.* Conversely, if the focal length of the eye-lens is too long, the lens is not convergent enough, and a parallel

beam will come to a focus beyond the retina. Using accommodation, the person can still see to infinity clearly, but will be limited at the near point when using the greatest convergence, possibly to somewhere beyond 10 inches, so such a person is called *long-sighted*.

The remedy in each case is to set a spectacle lens in front of the eye—or it may be stuck on the eye itself—so that an object at infinity is focused on the retina when the lens is in its normal state. Generally the calculation of the focal length of the added lens is done in terms of the *nearest distance of distinct vision*—that is to say, we apply the formula of p. 159 to the following problem. An object at x inches (inserting here the subject's nearest distance of distinct vision) is, by means of a lens at, or close to, the eye, to produce a virtual image at 10 inches; what focal length of lens is required? (Thus insert $u = -x$ and $v = -10$ in the formula and derive f in inches.) If the person is short-sighted, he requires a divergent lens to increase the focal length in combination with the cornea; if long-sighted, a convergent lens to reduce it.

Another defect which often supervenes with advancing age is lack of accommodation. The cornea gets set into a fixed shape, or the extent to which it can be bulged is much reduced. In this case the person must wear one set of spectacles to view objects at some distance from him and a more convergent set for reading and writing.

Convergent Lens Used to Form Virtual Image

The commonest application of a simple lens to give a virtual (enlarged) image is in the simple magnifying glass. The small object to be examined is held close to the lens, so as to fall within the focal length, and an enlarged image, the same way up, is then formed on the same side as the object (as in Fig. 58). This is, in fact, a simple microscope, and the magnification attained is estimated by the relative size of image to object as seen by the eye. The eye estimates the size of a body by the angle which it subtends at the eye, which in turn is defined by the width of the object (perpendicular to the direction in which it is viewed) divided

by its distance from the eye. Thus the magnification produced by a simple lens used in this way is limited, for while it is true that one can get a large image by putting the object close to the glass, the image will then be a long way off, since the relative size of image and object is the same as their relative distances from the lens (p. 160).

In practice the magnification to be attained by a simple lens rarely reaches one-hundred-fold.

Note that in order to see the whole of an enlarged image through the glass, the eye is put as close to it as possible. This is true of all microscopes and telescopes.

Optical System Formed of Two (or more) Lenses

1. The Compound Microscope.—In order to increase the magnification, two convergent lenses may be used, the first lens (called *objective*) nearer the object, O, giving an enlarged real image I_1, which acts as object for the second one (*eyepiece*) within its focal length, so that the final image I_2 is again enlarged and virtual.

The way in which the two images are formed is shown in Fig. 65. In practice both the lenses have a relatively small focal length. It must be remembered that the eye itself has a lens and that, other things being equal, a body looks largest to the unassisted eye when it is at the nearest distance of distinct vision. The same consideration applies to an optical instrument, so that one would naturally set the final image, I_2, at 10 inches from the eye (or slightly less, from the eyepiece). This will give some idea of the relative positions of lenses and images in the figure.

When it is desired to mark or measure the image a cross-wire or transparent scale is placed between the eyepiece and the objective at the position of I_1. This is then also magnified by the eyepiece lens and viewed at and alongside I_2.

The eyepiece usually consists itself of a combination of lenses, and so does the objective, but we have for simplicity shown each as single, as, in fact, it is in a cheap pocket microscope. It will be appreciated that when a large magnification without distortion is required, especially if it

is to cover a fairly large field of view of the object, great care has to be exercised in the fashioning of the lenses, just as in the making of camera lenses for the same purpose.

2. The Telescope.—The telescope consists of a similar pair of convergent lenses, and in principle acts on the light in the same way as the compound microscope. It differs only in the scale of the changes. A telescope is used for viewing distant objects. Consequently, the image I_1 in the objective is formed at its focus, and it will be diminished, but it will *look* bigger, being much closer to the eye than the distant object. In the eyepiece an enlarged virtual image I_2 of this is produced (as in Fig. 65). The

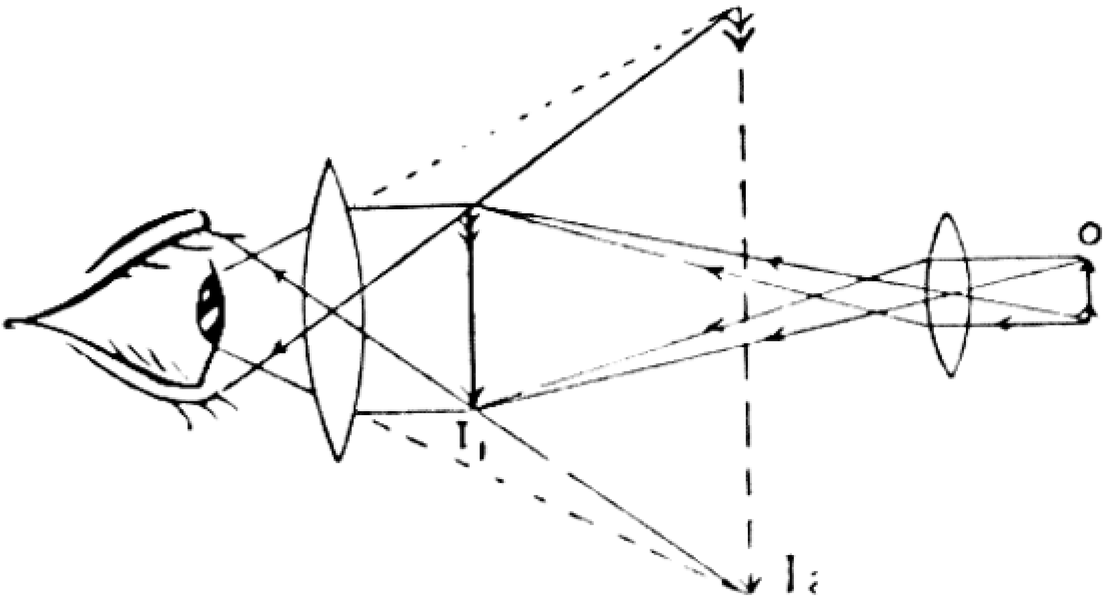


FIG. 65.—COMPOUND MICROSCOPE.

eyepiece then does not differ considerably from that of a microscope, but the objective, since it has to collect as much light as possible from the surrounding sky or land, has a wide aperture, whereas the objectives of powerful microscopes are only a few millimetres wide.

This form of telescope, as the figure shows, gives the final image upside down. This does not matter in an astronomical telescope, but in one used for terrestrial observation an additional lens (or lenses) may be placed in the barrel between the objective and the eyepiece. The course of development is then: (*a*) real inverted and diminished image in the object glass, (*b*) real re-inverted and, therefore, erect image (size practically unchanged) in the

intermediate lens, (*c*) virtual enlarged and still erect final image in the eyeglass.

The eyepieces and objectives of modern telescopes are also combinations instead of single lenses. In both microscopes and telescopes adjustment of the system for change in the object distance is done by moving either eyepiece or objective relative to the other in the barrel; in the telescope often by having each attached to lengths of tubes, one of which can slide over the other without letting in the light.

Instruments Involving Mirrors and Lenses.

A number of small instruments used for medical examination have the microscope as basis (either simple or compound), but with the addition of a lamp and a series of mirrors (and, possibly, auxiliary lenses) to illuminate the thing to be viewed. Owing to the restrictions of space, the axis of the illuminating system and of the microscope often have to coincide. Thus in the ' ophthalmoscope ' used for examining the surface of the retina a concave mirror reflects light into the eye, using the cornea itself to focus light onto the retina, while a small lens let into the central portion of the mirror enables the oculist to look at the illuminated portion of the retina. The ' laryngoscope ' for examining the throat and vocal cords is similar, except that an additional mirror on the end of a ' stalk ' is pressed against the back of the mouth to turn the beam of light through a right angle so that it illuminates the throat.

CHAPTER XIV

SOURCES OF LIGHT AND THEIR MEASUREMENT

Sources of Light

Since, as we have emphasized, differences in the production of heat and of light are mainly in degrees of temperature, most of the sources of heat which we have enumerated on p. 22 will serve equally well—by suitable adjustment of the temperature—as sources of light, provided the higher temperature necessary does not cause their decomposition. We shall, however, enumerate the common sources of light here, pointing out at the same time those which are suitable for standards. A standard source of light is one of which samples made at different places and at different times will, on taking adequate precautions, give their users the same light intensity.

1. The Candle.—In this venerable light-producer the rate of burning and the intensity of light are determined by the shape and the chemical nature of the tallow surrounding the wick. Candles made in a prescribed way were the original standards of light, hence a source is often described as having an output equal to so many times the standard candle or, shortly, as being of so many *candle-power*. The way in which the comparison is made is described later in this chapter.

2. Fuel Lamps; burning solid, liquid or vaporized fuel The fuel, if fluid, may be fed directly to a wick, as in the fast-disappearing oil lamp; or brought under pressure to a burner, as in lamps burning acetylene. A lamp burning pentane at a fixed rate (the Vernon Harcourt lamp) has also been used at various times as a standard.

If illuminating gas is fed to a filament or mantle coated with certain oxides, the latter become incandescent and add to the brilliance of the light. This is also a more economical way of producing light from the gas, since air can then be

mixed with it, the higher temperature so produced rendering the oxides luminous. The lime-light (now largely superseded by the arc-light) uses the same principle, in which the ignited gases are used to maintain a piece of ordinary lime at white heat. The brilliance of gas lamps is too closely affected by the rate of gas supply for use as standards.

3. Electric Lamps.—These are of three types.

A. *Filament Lamps.*—This is the commonest type. The filament of metal wire or of carbon burns in a vacuum or in an atmosphere free of oxygen, as this would, if present, burn the filament away. The filament is heated white hot by the passage of an electric current. With careful control of the temperature of the filament, such a lamp may be used as a standard. For every watt of electric power supplied to the average of such lamps an illuminating power of about two standard candles is produced.

B. *Arc Lamps.*—The original arc lamp burnt in a free supply of air, the arc being produced by touching together two electrodes of carbon through which a current was passing, and then separating them a small distance. The current continues to jump the gap, being conducted across by white-hot particles of carbon. Enclosed arcs can now be formed in atmospheres of other gases; for example, the mercury arc lamp, in which one of the carbon electrodes is replaced by a pool of mercury in an atmosphere from which the oxygen has been removed, is a prolific source of ultra-violet rays (see p. 182).

C. *Discharge Lamps.*—An intermittent electric discharge may be made to pass between two (cold) electrodes in various gases under low-pressure conditions approaching a vacuum. These are used in advertisement signs, as they consume very little electric energy. They are also used as spectral sources (see p. 179).

Measurement of Illuminating Power

An instrument in which an electric lamp, for example, can be compared as light-giving source with a standard is called a photometer.

The student can illustrate this work for himself by setting

out to compare his flash-lamp bulb with a candle, which he can regard for the nonce as his standard. One simple way of doing this is to set up a wooden cylinder—a pencil, for instance—in front of a white card in a darkened room and to put the two sources at some distance and facing the card, but not actually in line with the pencil. Each source acting alone would produce a black shadow on the card, but when both are alight there are two shadows of a greyish tinge. One shadow is, in fact, illuminated by source A and not by B, and the other by B and not by A. If the respective illuminations on the screen due to each acting alone are equal, the shadows will look equally grey, Move the candle until this is so. Now the light sent out from a source radiates in all directions, and so spreads according to the inverse square law (p. 105), so that if the distance of a

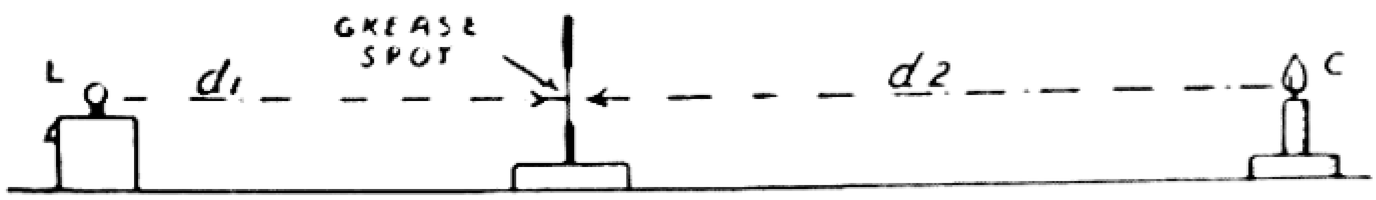


FIG. 66.—GREASE-SPOT PHOTOMETER.

screen from the source is doubled, the light falling on it is reduced to one-quarter of what it was when the screen was in the first position. Thus if, when the final adjustment is made, the candle assumed to be standard is found to be 1 foot from the screen, while the flash-lamp is at 3 feet from the screen, the latter must be $\frac{3^2}{1} = 9$ candle-power.

The above simple device is known as a shadow photometer. A better form, quite simply made, is called the grease-spot photometer (Fig. 66). Take a piece of tissue paper and spill a drop of thin oil into its centre. Take a post-card, cut a hole about 1 cm. diameter in its centre, and paste the tissue paper by its edges over the card, so that the ' grease spot ' covers the hole. If this be viewed with one's back to the light which is falling on the hole it looks darker than the card, because it lets the incident light through, whereas the main body of the card reflects light back into

the eye. From the other side, facing the light, the hole looks brighter because it is transparent. To make the photometer, set this card up vertically and hold it by a clamp on a table in a dark room. Put your lamp L and the candle C on the bench in line with it, but on opposite sides. Looking at one side of the card, the hole will appear brighter than the card, or the contrary, according as more light is coming through the grease spot from the other side than is falling on the card from this side. When the relative distances from the card are so adjusted that each source produces equal illumination on either side, the grease spot will be invisible (unless the colours of the two lights differ much).

When this condition of invisibility is secured, or most nearly approached, the *relative powers of the sources are directly proportional to the squares of their respective distances from the screen*, just as in the shadow photometer; $\frac{L}{C} = \frac{d_1^2}{d_2^2}$.

The difficulty of the difference in colour which may exist between two sources that have to be compared, for example, between a white electric lamp and a yellowish candle, may be met by presenting each portion of the card that is illuminated by each source alternately to the eye in rapid succession, like the succession of images in cinematograph projection. Under such circumstances it is found that the eye loses the perception of the slight colour difference, but is able to perceive a difference of intensity as a flicker in the picture presented to it.

CHAPTER XV

COLOUR

Production of Colour by a Prism

The phenomenon of refraction is not quite so simple as we have imagined it, so far. In the experiment with the pins and the prism described on p. 154, the student may have remarked that the refracted images of pins seen through the prism appeared coloured and with indistinct edges. Such an image is, in fact, not a single image, but a set of overlapping images, one for each tint in the infinite gradations of colour that make up white light.

If by means of a white source and a convergent lens, suitably 'stopped down' by black cards, a thin beam of

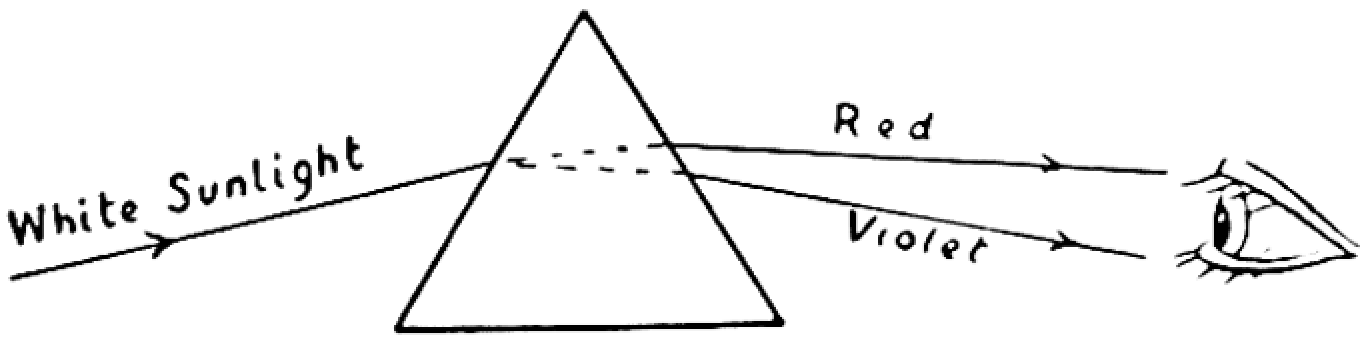


FIG. 67.—PRISMATIC COLOURS.

white light is cast on the prism, a whole series of colours can be found on a white screen placed on the other side (Fig. 67), which are the colours which make up white light. This is called a continuous spectrum, and it is seen at its purest—that is, with the least overlapping of the constituent colours—if the beam (or, at any rate, the central colour—yellow) passes through the prism in the symmetrical or minimum deviation position (p. 155).

From the order of the colours we shall observe that the *red has the least deviation and the violet the most.* In terms of our formula (p. 156) connecting the refractive index of the glass with the angle of deviation and the angle of the prism, this means that, for a given specimen of glass, or for a transparent liquid, the refractive index varies with colour,

being least for the red and greatest for the violet. On this account it is necessary to determine the refractive index of a specimen for a single colour—monochromatic light it is called—and yellow is usually chosen as being near the centre of the spectrum.

Separation of Colours by Transparent Media of other Shapes

It should be apparent to the reader that other shapes of glass bodies and of hollow transparent vessels containing

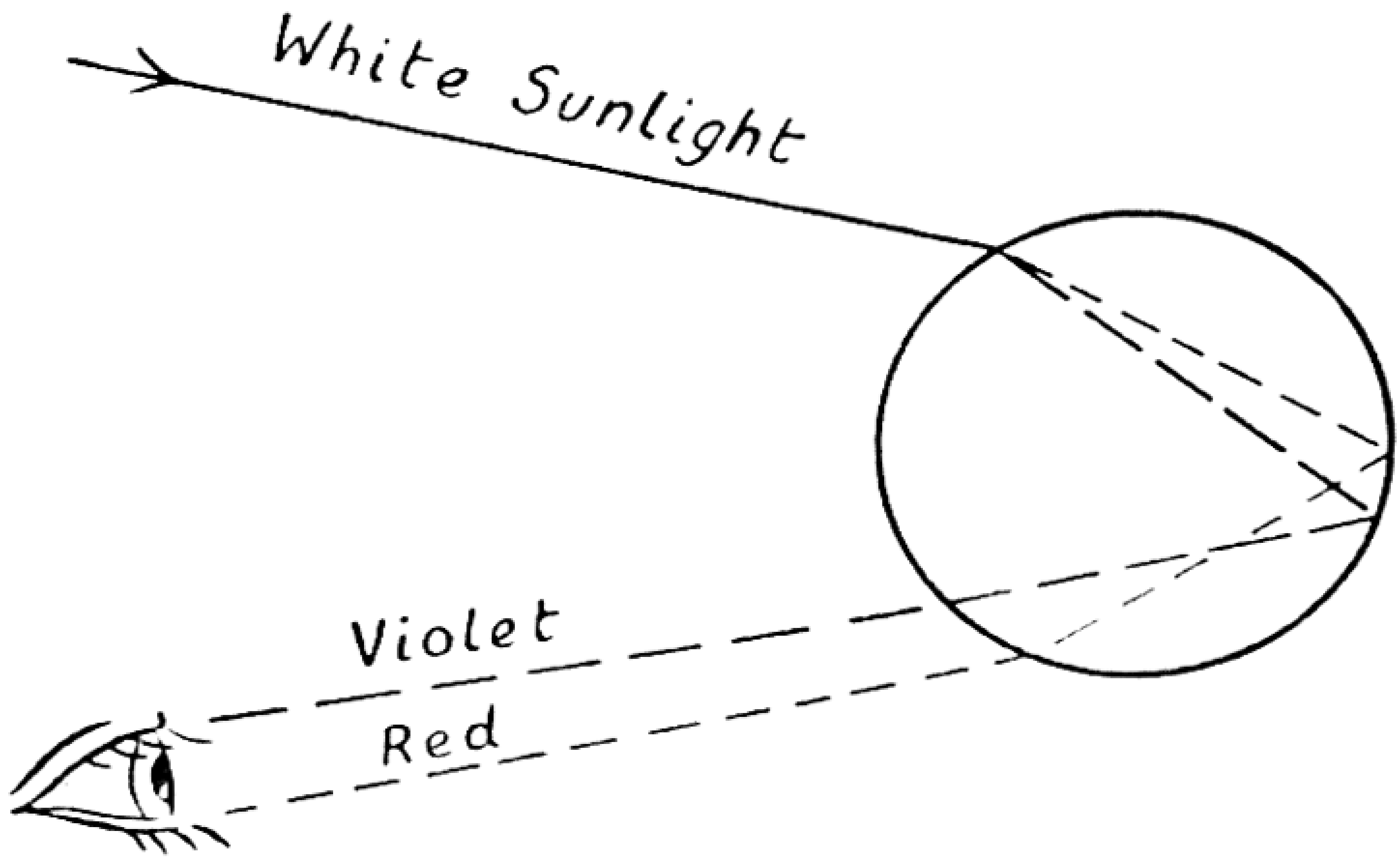


FIG. 68.—THE RAINBOW.

liquids can do to white light what the prism does. Only the parallel-sided slab or cell is immune in this respect, since it sends out a ray in the same direction to that by which it entered. The effect on other bodies is the greater as the angle between the faces becomes greater. In a lens this causes colouring of the edges of images formed by rays which have passed in and out near the rim, and has to be compensated by means which it is beyond the scope of this book to discuss.

A glass sphere or a spherical water-drop can separate out

the colours in sunlight by the unequal deviations produced. In this way the rainbow is formed by white light entering the raindrop, splitting up into colours on refraction, being reflected on the other side and out again by the side it entered, but turned down about 42° to its original direction. Fig. 68 shows the paths of the extreme (violet and red) rays. The eye placed where shown sees a bow in the heavens with the violet on top and the red below. Sometimes the light is further reflected to and fro before emerging—at a steeper angle—and giving rise to a 'secondary rainbow' seen above the first. As some light is lost at each reflection, this one is never so brilliant as the primary beam.

The Spectrum Emitted by Hot Gases

The continuous spectrum such as we have described is characteristic of white-hot solids. When the vapour of simple bodies is made to glow vigorously, usually a much simpler colour scheme is produced. Thus, if common salt is heated in a gas-burner and put in place of the white source in Fig. 67, there will be found on the screen just a line of yellow in the same place that the yellow stood when it was part of a continuous spectrum. This is known as a line spectrum—really, it consists of more than one line in most cases—and is due to the sodium in common salt. Line spectra may be got from the salts of metals in this way or from rarefied gases in tubes through which an electric discharge is made to pass.

Absorption Spectra

Many bodies have the facility of absorbing certain colours from white light which falls upon them. Thus a jar of red ink looks red when held up to the light, because the dye in it has the faculty of absorbing the other colours in sunlight—blue, green, yellow, etc.—leaving only the red to pass unmolested. If in the experiment of Fig. 67 we interpose a vessel containing the ink in the beam of light either before or after it has reached the prism, we shall find all our continuous spectrum disappear except the red end.

Even the sunlight does not reach us as a complete spec-

trum. Close observation of the solar spectrum shows a number of black lines indicating absorption of narrow lines of colour by the hot gases of the sun's atmosphere.

The Spectrometer

An instrument in which the examination of spectra may be carried out more accurately than with the simple apparatus already described is shown in Fig. 69, and is known as a spectrometer.

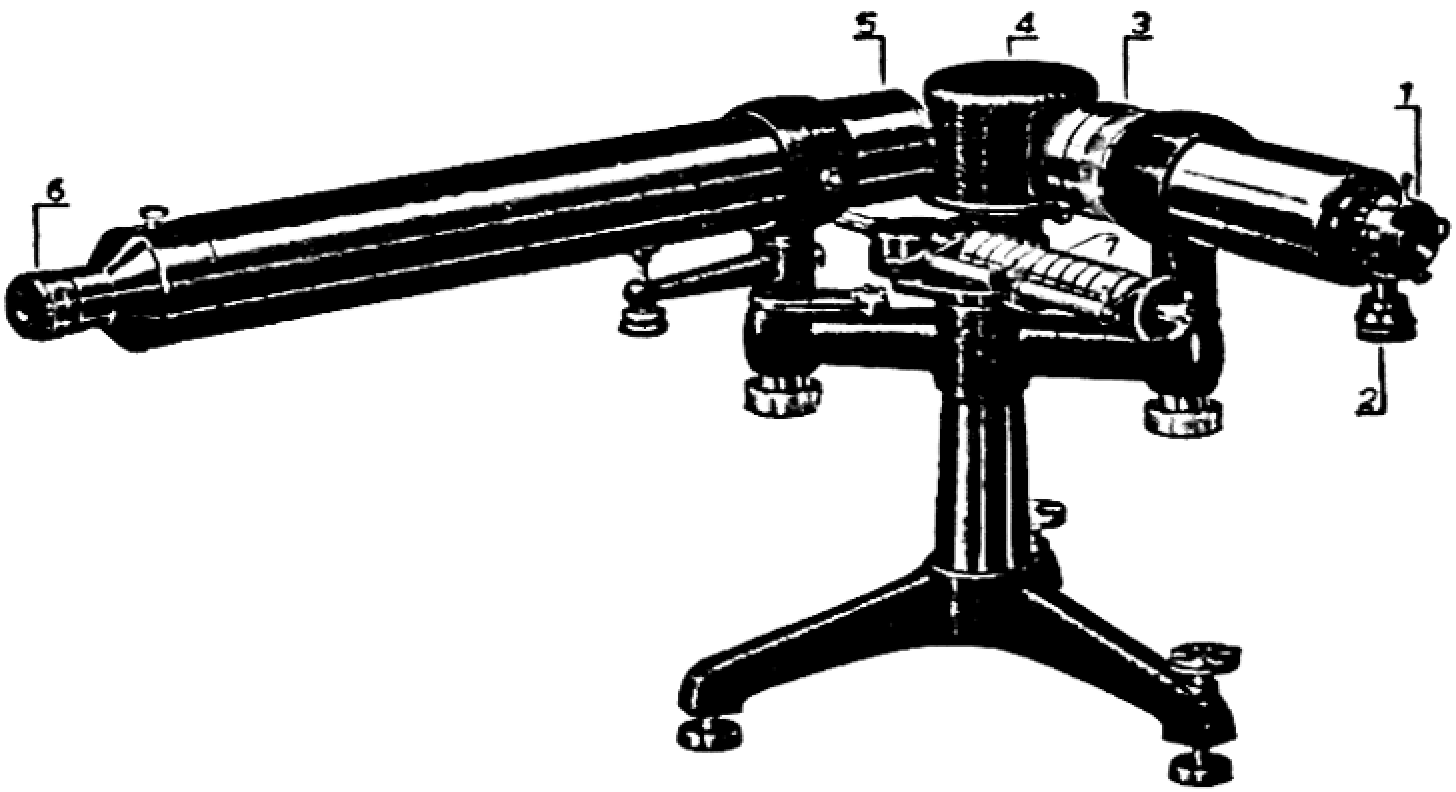


Fig. 69.—The Spectrometer.

The prism is placed on the turn-table shown at the centre, and the tube on the right-hand contains a convergent lens, at the focus of which the source is placed. This, called the *collimator*, delivers a parallel beam on one face of the prism, which, after emergence, travels through the *telescope* on the left through which the observer looks. The prism on its turntable can be turned round a vertical axis, its position at any moment being recorded on a circular scale below. Usually, the collimator and telescope can also turn about the same vertical axis through the centre of the table, their angular positions being recorded on another circular scale below. By this means the minimum deviation of any colour in the spectrum of the source may be read off on the spectro-

meter scale and—if the angle of the prism has also been measured—the corresponding refractive index may be calculated.

Continuation of Spectra beyond the Visible Portion

We have already remarked on the way in which an iron rod when heated first glows dully red, then scarlet, orange, yellow, and finally, at a sufficient temperature, white. Spectroscopic examination of the radiation which it emits discloses the fact that starting from the red end it gradually adds more and more of the continuous spectrum to its emission, until eventually the spectrum is complete and the colour white. The question now arises, what sort of radiation is it emitting before it shows any visible colour? It is natural to suppose that heat-rays have the same nature as light-rays, since we cannot have the latter from a solid without the former, but that only the portion limited by the continuous spectrum of this radiation can affect our eyes. The radiation which a hot body emits before its temperature is high enough to emit light is called infra-red. We can detect infra-red rays by their action on the skin, giving rise to the sensation of warmth, or by a thermometer. If, in the experiment with the prism of Fig. 67, we hold a sensitive thermometer so that its bulb is in the red region, we shall find it recording a rise of temperature. Moving the bulb a little beyond the red into the invisible region, an even higher temperature may be recorded at first, but further out the effect diminishes. We have therefore demonstrated the existence of rays beneath the red which are less deviated by the prism than any of the visible rays.

Photographic plates can be made which are sensitive to infra-red rays, so that it is now possible to photograph the invisible. Since, too, these rays penetrate a fog more readily than light-rays, such plates are useful for general photography under misty conditions.

If an ordinary photographic plate is exposed to the continuous spectrum by setting it up in place of the eye of Fig. 67, it is found that there is some radiation which affects it beyond the visible extreme violet end. These are called

ultra-violet rays. We have no sense by which we can detect these, but—besides their reducing action on chemical compounds in the photographic emulsion—one can demonstrate their presence by the *fluorescence* which they induce in certain bodies. A fluorescent substance is one which, absorbing radiation of one type, re-emits it as another type. Such substances, shone upon by invisible ultra-violet rays, emit greenish or yellowish visible rays. A certain liquid—which, from its possession of this property, is known as 'fluorescein'—can be held in a small phial beyond the violet end of the spectrum and seen to give out a green light.

More copious ultra-violet rays can be obtained from an electric arc formed in an atmosphere of mercury (see p. 174). The spectrum of mercury vapour consists of certain lines in the blue and violet and more in the ultra-violet. The former can be cut off by certain coloured glasses, so that, seen through such a filter, no light coming from the lamp is visible. However, the fact that ultra-violet rays are getting through can be demonstrated with the aid of the phial of fluorescein, or by leaving a photographic plate near the lamp in a darkened room and subsequently developing the plate.

Because glass absorbs ultra-violet rays fairly readily, it is advisable to substitute a prism of quartz (which lets them through) for the prism of glass when carrying out the experiment above with the fluorescein held beyond the violet end of the spectrum. For the same reason, the bulb containing mercury in which the mercury arc is struck is made of fused quartz (silica) instead of the glass used in other lamps.

CHAPTER XVI

HEAT AND LIGHT AS WAVE MOTION

We have already hinted that heat and light may be regarded in most of their manifestations as a series of waves sent out from the source and passing through all transparent bodies. The fact that sound and light obey the same laws of propagation (inverse square law), reflection and refraction suggests this, but with the important difference that sound involves the material movement of particles, whereas radiant heat and light do not. Sound cannot travel through a vacuum, but heat and light do so reach us from the sun.

From the observed fact that radiant heat, light and 'radio' waves travel with the same velocity through a vacuum—namely, 300,000 kilometres per second—we infer that all three waves are of the same type, and involve some sort of electric and magnetic displacement. Without attempting to go further into this difficult question of the nature of the displacement or disturbance involved, we propose in this final chapter to point out some of the consequences of the assumption that heat and light are vibrations, propagated as waves.

We have seen (p. 102) that a progressive wave has associated with it a wave-length and a frequency of vibration. The product of these quantities is the velocity (*in vacuo*). Without attempting to explain how these are measured, we shall give a figure (70) showing how the wave-length progressively decreases from the infra-red through the visible spectrum to the ultra-violet, while the frequency, of course, increases. (On the same figure we have indicated the region which is predominantly associated with heat-rays.)

Now, when these rays enter a dense but transparent medium like water or glass, their velocity (as waves) decreases, and it turns out that the ratio of the velocity *in vacuo* to that in the substance is equal to the refractive index for the substance to those waves. Since violet rays are

more refracted than red, we conclude that the index of refraction is greater for them, or greater as the wave-length diminishes.

The aspects of light which concern wave motion are in no respect more brought home than in diffraction. In the Vibration and Sound Section we pointed out that when waves encounter an obstacle or an aperture, the extent to which the waves are diffracted or spread from the straight path beyond is governed by the ratio of the size of the obstacle or aperture to the wave-length, and we drew a figure (41) to illustrate this. Now, sound-waves averaging several feet in wave-length commonly encounter objects and

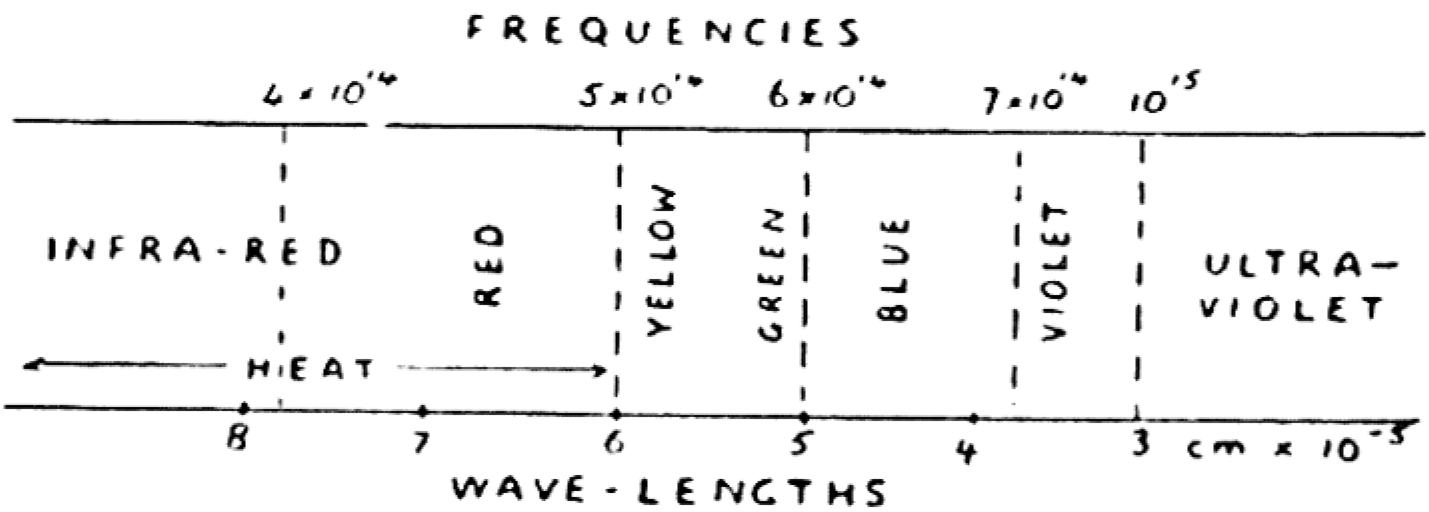


FIG. 70.—THE SPECTRUM.

openings which are small compared to the wave-length, so that sound is readily diffracted and heard round corners, etc. Light, on the other hand, has such a small average wave-length (see Fig. 70) that it appears to travel straight forward, giving direct beams through holes and sharp shadows of obstacles. Only when the obstacle or aperture is less than a millimetre in width can diffraction be demonstrated. The shadow of a thin needle, for example, is not clear-cut, particularly in white light, where the shadow is covered with overlapping diffraction bands of different colours. This is because the pattern of the diffraction depends on the ratio of the wave-length to the size of the needle, a factor varying from colour to colour.

Diffraction of light by small obstacles is also apparent in the effect of a fog, which is a collection of small water

droplets about one hundredth to one thousandth of a millimetre in diameter, on white light. The red and the infra-red rays, though absorbed to a certain extent, still manage to penetrate unless the fog is thick, but the blue and violet are so diffracted and scattered by these droplets which approach their wave-length in size that little of this radiation gets through to the other side. Consequently white lamps and the sun seen through a fog appear orange or red in hue.

CHAPTER XVII

MODERN DEVELOPMENTS AND APPLICATIONS

Infra-red Rays

The existence of these rays was mentioned in the previous chapter. In order of increasing wave-length, they follow visible light in the electromagnetic spectrum. Herschel discovered this type of radiation in 1800, by passing sunlight through a prism, and then placing a thermometer in the invisible region of the spectrum just beyond the red. Using other thermometers out of reach of the sun's spectrum as a check, he was able to show a rise in temperature in the invisible region beyond (that is of greater wave-length than) the red. He at first regarded these rays as being entirely different from visible light rays, but about 1840 it became generally accepted that they were of the same nature as light and differed from it only in wave-length.

The infra-red region is quite properly regarded as beginning just beyond the longest red wave-length, which the eye can see. This has a value of about $7{\cdot}6 \times 10^{-5}$ cm. A more commonly used unit for these wave-lengths is the micron, equal to 10^{-4} cm., and written as $1\,\mu$. Hence, using this unit, the lower limit of wave-length just mentioned is $0{\cdot}76\,\mu$. The upper limit is about $400\,\mu$ (that is 4×10^{-2} cm.). Whereas visible light extends over about one octave, the infra-red or heat-ray region occupies about nine octaves.

In principle, infra-red rays can be detected and their wave-lengths measured by an instrument similar to the spectrometer shown in Fig. 69. It must, however, require some modification in view of the fact that these rays cannot be seen by the naked eye, and further that prisms made of glass absorb infra-red rays if their wave-length is longer than $3\,\mu$. The first difficulty is overcome by using electrical methods of detection instead of the eye. Thus if the infra-red radiation is allowed to fall on a thin blackened strip of platinum, the platinum will be heated and its electrical resistance will increase. This increase can be detected by

connecting the platinum strip as one arm of a Wheatstone bridge circuit. (For particulars of this circuit, see *Teach Yourself Electricity*.) A strip connected in this way is known as Langley's bolometer. Another method is to use a thermopile as detector. This consists (Fig. 71) of a number of strips of two different metals (often bismuth and antimony) connected alternately in series. Alternate junctions of the two metals are exposed to the infra-red rays and are heated by them. The remaining junctions are screened from the rays and are kept at room temperature or in ice. On connecting a sensitive galvanometer across

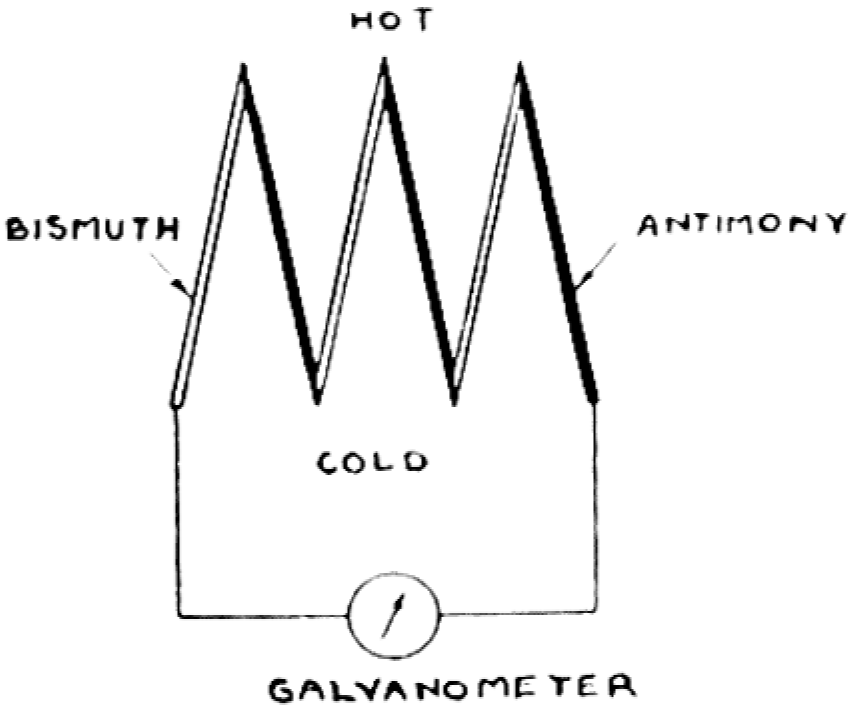


FIG. 71.—THERMOPILE.

the ends of the composite wire (as shown in Fig. 71) a small current is found to be flowing, although there is no battery in the circuit. This is produced only when there is a difference of temperature between the two sets of junctions. Since this difference is caused by the infra-red rays the apparatus acts as a sensitive detector of them.

The most sensitive detector of these rays within the wave-length range of 1–3 μ is the lead sulphide cell. This consists of a small glass plate on which is deposited a thin film of lead sulphide. This film is often covered by an extremely thin layer of lacquer which protects it from dust and fumes, while allowing the infra-red rays to reach it. Opposite ends of the lead sulphide film are connected to terminals

and its resistance measured in a Wheatstone bridge circuit. It is found that the resistance of the film is considerably and rapidly altered when infra-red radiation falls on it. If this radiation comes from a distant source and is weak, its detection can be made easier by using a concave mirror, as shown in Fig. 54(*a*), to focus the rays on to the cell. Further the effect of the resulting resistance change may be greatly magnified by the use of a valve amplifier.

The wave-length of these rays may now be found by passing them through a prism, of known refractive index, and detecting them by one of the three methods mentioned. If the rays are longer in wave-length than 3 μ, they will be absorbed by a glass prism. However, prisms of rock salt or potassium iodide may be substituted. These transmit wave-lengths up to 15 and 30 μ respectively. An instrument used for measuring infra-red wave-lengths, consisting of a suitable prism and detector, is called an infra-red spectrometer.

The practical applications of infra-red rays are already numerous, and undoubtedly many more remain to be discovered.

One of the first applications was made in photography. It was explained in Chapter XVI that light waves are diffracted and scattered by small particles. Even the molecules composing the atmosphere scatter light, as well as any small droplets of water or smoke particles which may be present in a haze. An impression of light scattering is given in Fig. 72. It occurs to a smaller and smaller extent the longer the wave-length of the light being used. Thus blue light is scattered much more than red light, and red light much more than infra-red rays. Hence infra-red rays can penetrate long distances through the atmosphere, or even through a haze, with less diminution in their intensity than ordinary light. This clearly has application in the photography of objects at long distances or slightly obscured by haze. Until 1919, photographic plates were not sensitive to the infra-red, but since that time and especially since 1934, the sensitive range has been extended more and more into the infra-red. It is now possible to

obtain plates actuated solely by the infra-red, the visible rays being prevented from entering the camera by the use of a suitable filter. This great advance has made possible long-distance photographs, in great detail, of subjects such as Calais taken from Dover, and Mount Everest from Darjeeling—more than one hundred miles away! It should be emphasised, however, that infra-red rays will not penetrate very thick fog.

Still greater wonders were developed during the 1939–45 war. German scientists invented a method of floodlighting

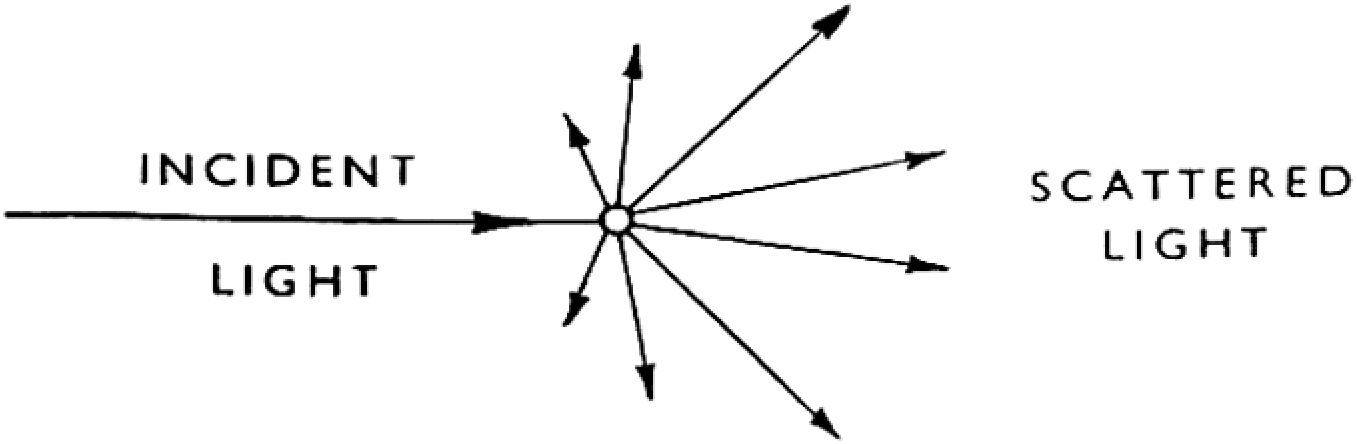


FIG. 72.—SCATTERING OF LIGHT.

an area, to be observed at night, with infra-red rays and then converting the reflected infra-red picture into visible light, so producing a clear image under conditions of total darkness! The source of light was a tungsten-filament lamp with a filter in front, allowing only infra-red rays to pass through it. After reflection from the objects to be viewed, the returning rays were received in a special tube shown in section in Fig. 73. They were focused by a lens, the objective, on to a very thin film of a rare metal, cæsium, forming an infra-red picture there of the scene being viewed. The infra-red rays caused the cæsium to give off electrons, tiny negatively charged particles, the number of electrons given off by any part of the picture depending on the amount of infra-red energy received by it. These electrons were then attracted towards a positively charged metal tube, a negative charge being attracted by a positive charge. They passed through the tube, which focused the electron beam much as a convex lens focuses light, and were further

attracted to a thin fluorescent screen, similar to the screen of a television set. There the impact of the electrons caused the emission of light, building up a clear picture in visible light of the area originally illuminated only by invisible infra-red rays! This system was used by the Germans for tank attacks in total darkness, and a smaller version was attached to rifles for use by snipers. There would seem to be a future for an infra-red device to give warning of obstructions in the path of trains and small

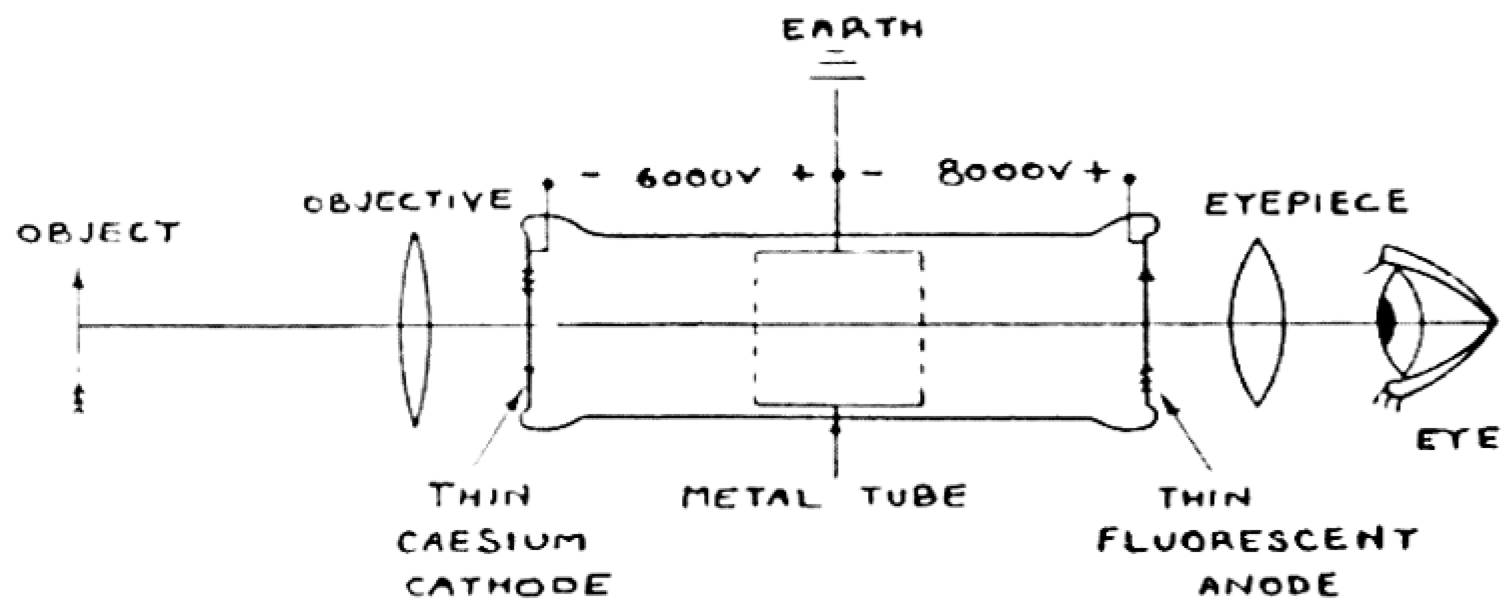


FIG. 73.—GERMAN INFRA-RED TELESCOPE.

ships. It should be possible to make it smaller and cheaper than its radar equivalent, though it would not work in very thick fog.

Infra-red lamps are now finding many uses. First designed for the relief of rheumatism and neuritis, their heat-giving rays penetrate the skin and give warmth to the underlying muscles and tissues. Specially adapted lamps are now much used for processes involving the evaporation of water or the drying of paints and lacquers. This was often done in the past by the circulation of hot air, that is by convection, but it can now be done more quickly and cheaply by radiation (see Chapter V) using infra-red rays. This is because the transfer of heat from one medium to another (say air to water) by convection and conduction is always made difficult by a resistance at the surface dividing the two media. Thus the heat-transfer coefficient

between air and water is low. Again when the heat enters the water its movement is made difficult by the low heat conduction of water. Similar arguments apply to the drying of paint on a surface. In transferring heat by radiation from an infra-red lamp, however, the air round the lamp is not necessary and has little effect. Further, when the rays strike the water or paint they can penetrate some distance before they are fully absorbed. The drying process therefore goes on in a layer of the material and not merely at the surface. The bad effect of the poor heat conduction of the material is minimised, and drying is accelerated. The lamps used have tungsten filaments burning at lower temperatures than normal electric lamps. They often incorporate their own reflector by having a thin coating of aluminium deposited on the inside surface of the glass envelope. They give off a band of infra-red rays, the wave-length of maximum intensity being about $1{\cdot}2\,\mu$. Banks of these lamps are now much used in motor-car and aircraft factories for drying paint and lacquer. They are also used in textile mills, where it has been found possible to dry yarn in one-quarter of the time formerly required, without any harmful effect on the material. The drying of vegetables, fruit, tobacco and even developed photographic films has been accelerated by the same means. It seems possible that grass drying, at present carried on by farmers using coke or oil heating, may eventually be done by infra-red methods.

Sounds which Cannot Be Heard

The ear is a remarkable instrument, to a degree few appreciate. Those of average sensitivity respond to sounds of frequency (p. 94) from about 16 to 16,000 waves per second. Young persons can sometimes hear sounds of frequency as high as 20,000 per second, but this upper limit gets smaller with increasing age. The ear is most sensitive to sounds of frequency about 3000 per second. At this pitch it can detect air movement whose amplitude (p. 94) is as small as 10^{-10} cm. This is only one ten thousand millionth of a centimetre! Little surprise may be excited

until it is pointed out that it is less than the diameter of the hydrogen atom, one of the smallest things in nature, quite invisible to the most powerful optical microscope! Indeed, if the ear had been made more sensitive it would have been disturbed by the impact of the air molecules, which are always in ceaseless motion. It has been calculated that when listening to the faintest audible sound, the sound energy received by 1 square centimetre of the eardrum is equal in amount to the light energy received by the same area of the retina (p. 167) of the eye from a candle 8 miles away, in clear air. Moreover, the eye would not be able to see a candle at such a distance, so that the ear is a more sensitive receiver than the eye!

This introduction will make it the more surprising that there are sounds which, though intense, cannot be heard by the human ear. These are the sounds of frequency greater than 20,000 waves per second, which are called ultrasonics. It was discovered by the brothers Curie in 1880, that if a plate is suitably cut from a quartz crystal and electric charges of opposite sign applied to opposite faces, the dimension joining them will either increase or decrease. Consequently if these charges are applied and then reversed at a steady rate, the plate will pulsate. Further, we know (p. 113) that if forces are applied to a body at a rate equal to one of its natural frequencies, the amplitude of movement of the body will be large. It will be said to resonate. Clearly if an alternating voltage could be applied to opposite sides of a quartz plate and its frequency varied until one of the plate's natural frequencies was reached, the plate could be made to resonate with the frequency of the applied voltage. It would become a source of sound of that frequency. When the Curies discovered this effect no source of alternating voltage of widely variable frequency was readily available, but with the invention of the valve oscillator this was changed. Good-quality quartz is becoming scarce, but it has been found possible to grow, artificially, suitable crystals of other substances such as barium tartrate and ammonium dihydrogen phosphate (sometimes called A.D.P.), which are good substitutes. All

crystals with this property are said to be piezo-electric. It is also possible to make rods of magnetic materials, such as iron and nickel, vibrate when placed inside a coil fed with alternating current of appropriate frequency. Such a process is called magnetostriction. Thus ultrasonic waves can be produced in at least two ways. In practice, they range in frequency to-day from 20,000 to about 10 million vibrations per second. Sounds of these frequencies not only cannot be heard, they travel in narrow beams instead of spreading out as ordinary sounds do. This is because since the frequency is high, the wave-length must be short, wave-length being inversely proportional to frequency (p. 102). Hence if a crystal is of 2 or 3 centimetres diameter, this dimension may equal several wave-lengths. This is the condition required for a narrow beam (p. 106). It is not easy to obtain single crystals much larger than this, but, by using a number of such crystals in the form of a mosaic, a composite crystal of large diameter can be made and an almost parallel beam of ultrasonics produced. Such a beam found an immediate use in echo-sounding at sea. Knowing the velocity of sound in sea-water, a short pulse of ultrasonics is sent out from a transmitter in the bottom of the ship's hull, and the time taken for it to return, after reflection from the sea-bed, is measured. So the distance it has travelled in this time can be found. This is clearly equal to twice the distance from the bottom of the hull to the sea-bed, and so the depth of water under the ship can be found. In practice, the computation of the depth is carried out automatically and registered on a chart, no calculation being required of the operator. Further, it is not necessary for the ship to slow down or stop, as is essential when the old-fashioned leadline is used. When the apparatus is left running continuously, a profile of the sea-bed will appear on the chart. If a ship is travelling regularly between two ports, and such a profile is made in clear weather when its position is accurately known throughout the voyage, the result is of great assistance when fog is encountered on a subsequent voyage. Echo-sounders are also much used by trawlers in locating fishing banks. It

is also possible to detect shoals of fish, and some skippers even claim to be able to identify the species!

Ultrasonics are proving of great value in locating pin-holes and cracks in metal castings. An ultrasonic beam is directed along the length of the casting (Fig. 74) by the transmitter. Any reflected energy is picked up by the receiver. Some of the beam striking the end B of the

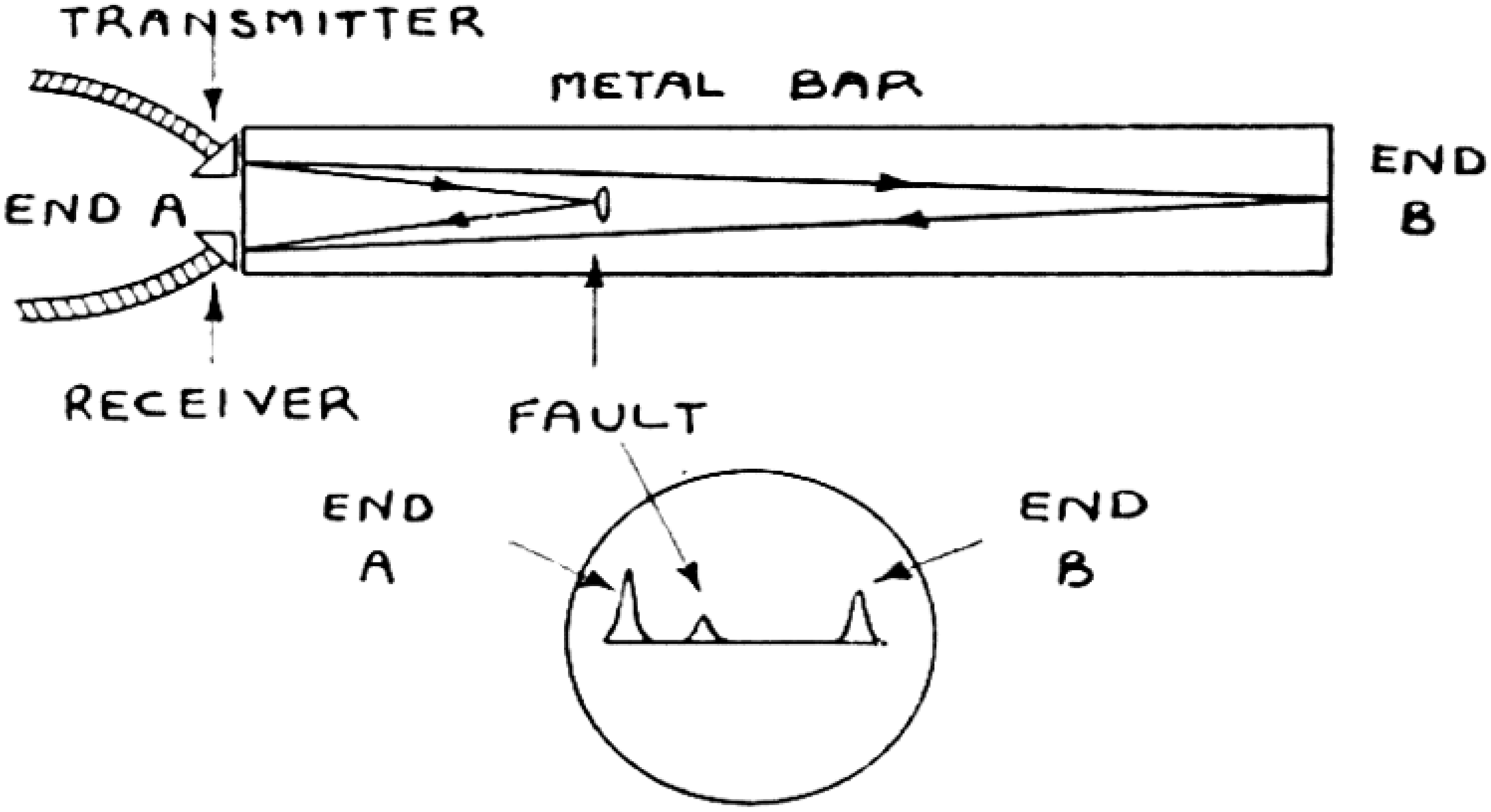


FIG. 74.—LOCATION OF A FAULT IN A METAL BAR BY ULTRASONICS.

casting is reflected. In addition, any sudden change in the character of the metal, such as a crack or other fault, will reflect some of the energy striking it. The receiver is moved over the surface of the end A of the casting until it also picks up the reflected beam from the fault. The method of indicating the position of the fault was inspired by principles used in radar. On a cathode-ray tube the position of the transmitter on the end A of the casting is represented by a spike of light, and so is the point of reflection of the beam at the end B. The distance between these two spikes therefore represents the length of the casting to a scale which can soon be calculated. Any spike appearing between these is due to a fault in the metal, and knowing the scale of the picture on the tube, its distance along the length of the casting can be found. If now the

transmitter and receiver are moved so that the ultrasonic beam traverses first the breadth and then the depth of the casting, the position of the fault can be located accurately. Ultrasonics will penetrate a much greater distance into a casting than will X-rays, and the apparatus required is cheaper and can be carried in a suitcase.

If ultrasonic waves are directed into molten metal just before it is poured into moulds, any gas bubbles present are removed by the vigorous agitation given to the metal. As a result, the castings produced are free of gas holes.

Ultrasonics are being used in cosmetic manufacture because of their aid in the formation of emulsions and in the mixing of creams, both processes proceeding much more readily under ultrasonic irradiation.

Though inaudible to humans, certain whistles operating in the lower-frequency ultrasonic range are readily responded to by dogs. It is possible that some insects make high-frequency sounds, inaudible to us, and it has been shown conclusively by American physicists that bats emit ultrasonic waves in flight and rely on their reflections to assist them in avoiding obstacles. The normal frequency of their high-pitched beam is about 50,000 waves per second. They appear to judge the distances of obstacles by the time taken for an emitted pulse of sound to return after reflection. This is the basis of radar methods of distance measurement. When objects have to be investigated at short range, it is usual in radar to use still higher frequencies than at longer ranges. It is therefore interesting to find that when very near to an obstacle, bats have been observed to use the same device of increasing the frequency of their "sonar" apparatus. It is surely remarkable that a technique only recently developed by man and rightly acclaimed as almost miraculous, has been in use by lowly creatures for millions of years!

New Uses for Heat

In many countries successful agriculture requires irrigation water, and this often comes from snowfields. Thus, agriculture in the northern regions of India and West Pakistan

is much dependent on water originating in the snows of the Himalayas. Snowfall amounts vary from season to season, so that the resulting water supply is also variable. Recent work on irrigation schemes in California has shown that these variations can be reduced by applying a knowledge of radiation (p. 85) and conduction (p. 75). Thus by scattering coal dust on snowfields, their ability to absorb the sun's heat is increased, with a marked rise in the resulting flow of water. Coal dust of little value as a fuel is effective and can be spread from a helicopter. If the water flow is excessive, this can be controlled by distributing sawdust (a bad conductor) on the snow, which can thus be conserved for the next year.

Far from mountain snows, rain is the principal source of water, but frequently a most unreliable one. Moist winds often pass over a dry area without producing rain. Raindrops begin as minute droplets, and it is believed that these can be induced to form in moist air if nuclei are supplied to act as collecting points. Dust particles behave in this way, and so do small crystals of silver iodide. Attempts at 'seeding' clouds with these crystals have been carried out in many parts of the world in efforts to produce rain. The crystals are usually thrown from an aircraft, but occasionally they have been carried upwards in rising air currents from fires. It is believed that these experiments have sometimes been successful in causing rain, but caution is necessary in their interpretation, since it might have rained without their aid.

Fog has been an enemy of aviation ever since regular schedule flights began. During the Second World War, successful efforts to disperse fog over aircraft runways were made by the system called 'Fido'. In this, petrol vapour, emitted from nozzles, was burnt. The resulting heat raised the temperature of the atmosphere in its neighbourhood, reduced its relative humidity (p. 71), and dispersed, or at least diminished, the fog. To have a worthwhile effect, many thousands of gallons of petrol had to be used, so that the cost could be justified only in the exceptional circumstances of war. Experiments are now proceeding to make

'Fido' more economic. Instead of petrol, cheaper fuels, such as gas oil, are being tried, and it has been realised that heating can be concentrated mainly in the approaches to the runway. When an aircraft reaches the threshold of the runway it is then usually possible for the pilot to see by the improved lighting now in use. This distribution of heat is achieved by having the burners close together on the approaches to the runway and farther apart along the runway itself, where the aircraft will be lower. In a cross-wind fuel economy is gained by using only the burners on the windward side of the runway. These improvements, which may allow an aircraft to be landed in fog at only one-tenth of the cost of the original 'Fido' system, should constitute a great advance in all-weather commercial aviation.

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